Introduction of Renewable Energy Technologies

Lecture-08
Solar Angle and Estimation of Solar Radiation

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Recap of the last lecture

- Apparent motion of the Sun
- Declination angle between equatorial plane and plane of revolution
- Motion of the Sun at given latitude throughout the year
- Optimum angle of solar collector
In this lecture

- Solar radiation on collector surface
- Local apparent time
- Sunrise, sunset time and day length
- Estimation of Solar radiation at any given time and at any surface
Solar Radiation on Collector Surface
Solar radiation at the collector

- The amount of radiation falling on a collector surface depends on the angle between the sun ray and perpendicular to collector

\[ I = I_b \cos \theta \]

- Generally the collector should be perpendicular (\( \theta = 0^\circ \)) to the solar radiation at all times

- But this requires continuous sun tracking
The parameters that can affect the direction of light reaching a surface are:

- Latitude of location ($\phi$)
- Day of year ($\delta$)
- Time of the day ($w$)
- Inclination of surface ($\beta$)
- Orientation in horizontal plane ($\gamma$)

$\theta$ is affected by five parameters.
Angles related to solar geometry

**Latitude** $(\varphi)$ – angle of a location on earth w.r.t. to equatorial plane
Surface azimuth angle $(+90^\circ$ to $-90^\circ$, +ve in the north)

**Surface azimuth angle** $(\gamma)$ – angle between surface normal and south direction in horizontal plane, $(+180^\circ$ to $-180^\circ$, +ve in the east of south)

**Hour angle** $(\omega)$ – angular measure of time w.r.t. noon (LAT), $15^\circ$ per hour, $(+180^\circ$ to $-180^\circ$, +ve in the morning)

**Surface slope** $(\beta)$ – Angle of the surface w.r.t horizontal plane $(0$ to $180^\circ)$

**Declination angle** $(\delta)$ – Angle made by line joining center of the sun and the earth w.r.t to equatorial plane $(+23.45^\circ$ to $-23.45^\circ)$
Incidence angle of rays on collector (w.r.t. to collector normal)

\[
\cos \theta = \sin \phi (\sin \delta \cos \beta + \cos \delta \cos \gamma \cos \omega \sin \beta) \\
+ \cos \phi (\cos \delta \cos \omega \cos \beta - \sin \delta \cos \gamma \sin \beta) \\
+ \cos \delta \sin \gamma \sin \omega \sin \beta
\]

**Case-1:** i.e. \( \beta = 0^\circ \). Thus, for the horizontal surface, then:

\[
\cos \theta = \sin \phi \sin \delta + \cos \phi \cos \delta \cos \omega = \cos \theta_z
\]

**Case-2:** \( \gamma = 0^\circ \), collector facing due south

\[
\cos \theta = \sin \delta \sin (\phi - \beta) + \cos \delta \cos \omega \cos (\phi - \beta)
\]
What will be the angle of incidence on vertical surface facing due South?
Local apparent time (LAT)

Time at which the Sun becomes overhead at a given location

• Normally the standard time for a country is based on a noon (overhead Sun position) at a particular longitude

• Correction in the real noon time by considering the difference in the longitude w.r.t. standard longitude of that country, \(1^\circ\) longitude difference = 4 min.

\[
LAT = T_{local} + \frac{1}{15}(Long_{st} - Long_{local}) + \frac{1}{60} \text{ Eq. of time}
\]
• Due variation in earth’s orbit and its speed of revolution
• Based on experimental observations
Local apparent time (LAT)

Local apparent time,

\[
LAT = IST - 4(\psi_{STD} - \psi_L) + E
\]

\[
E = 9.87 \sin 2B - 7.53 \cos B - 1.5 \sin B
\]

\[
B = (n - 81) \frac{360}{364}
\]

Determine the local apparent time (LAT) corresponding to 1430h at Mumbai (19°07’ N, 72°51’ E) on July 1. In India, standard time is based on 82.5°E.
Local apparent time, $\text{LAT} = \text{IST} - 4(\psi_{STD} - \psi_L) + E$

$E = 9.87 \sin 2B - 7.53 \cos B - 1.5 \sin B$

$B = (n-81) \frac{360}{364}$

$n = 182; B = 99.89; E = -3.524$

Local apparent time = 1430h – 4(82.50-72.85) minutes + (-3.524) min

= 1430h – 38.6 minutes - 3.5 minutes

= 1430h - 42.1 minutes

= 13h47.9 min = 1348h
Estimation of sunrise and sunset time, day length
## Terminology

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day number of the year</td>
<td>( n )</td>
<td>number</td>
</tr>
<tr>
<td>Latitude</td>
<td>( \varnothing )</td>
<td>degrees</td>
</tr>
<tr>
<td>Longitude</td>
<td>( \Psi )</td>
<td>degrees</td>
</tr>
<tr>
<td>Elevation</td>
<td>( E_L )</td>
<td>km</td>
</tr>
<tr>
<td>Declination angle</td>
<td>( \delta )</td>
<td>degrees</td>
</tr>
<tr>
<td>Slope of the collector</td>
<td>( \beta )</td>
<td>Degrees</td>
</tr>
<tr>
<td>Hour angle</td>
<td>( \omega_s )</td>
<td>Degrees</td>
</tr>
<tr>
<td>Number of sunshine hours</td>
<td>( N )</td>
<td>Hours</td>
</tr>
<tr>
<td>Maximum number of sunshine hours</td>
<td>( S_{\text{max}} )</td>
<td>Hours</td>
</tr>
<tr>
<td>Monthly average daily global radiation</td>
<td></td>
<td>kW/m(^2)-day</td>
</tr>
<tr>
<td>Monthly average daily diffuse radiation</td>
<td></td>
<td>kW/m(^2)-day</td>
</tr>
<tr>
<td>Total daily radiation falling on a tilted surface</td>
<td></td>
<td>kW/m(^2)-day</td>
</tr>
</tbody>
</table>
Day number of the year, $n$

$n$ varies from 1 to 365

For example: Jan 1$^{st}$, $n = 1$, and so on.

<table>
<thead>
<tr>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>28</td>
<td>31</td>
<td>30</td>
<td>31</td>
<td>30</td>
<td>31</td>
<td>31</td>
<td>30</td>
<td>31</td>
<td>30</td>
<td>31</td>
</tr>
</tbody>
</table>

Declination angle

$$\delta = 23.45 \times \sin \left[ \frac{360}{365} \times (284 + n) \right]$$
Sunrise and Sunset time

- Solar incidence angle for horizontal collector

\[
\cos \theta = \sin \phi \sin \delta + \cos \phi \cos \delta \cos \omega = \cos \theta_z
\]

For sunrise as well as for sunset the \( \theta = 90^\circ \) in above equation
Sunrise and Sunset time

• Hour angle at sunrise or sunset as seen by observer on an horizontal surface due south ($\gamma = 0^\circ$) will be

$$\omega_s = \cos^{-1}(-\tan \phi \tan \delta)$$

This equation is valid for day under consideration lies between September 22 and March 21 and location is northern hemisphere (due to negative declination).

• If the day of consideration is lies between March 21 and Sep 22, the equation is

$$\omega_{st} = \cos^{-1}\left[-\tan (\phi - \beta) \tan \delta \right]$$
The magnitude of hour angle corresponding to sunrise or sunset for an **inclined surface facing south** \((\gamma = 0^\circ)\) is

\[
|\omega_{st}| = \min \left[ \arccos \left( - \tan \phi \tan \delta \right), \arccos \left( - \tan (\phi - \beta) \tan \delta \right) \right]
\]

The sunrise for the inclined surface will happen later than horizontal surface for dates between 22 March o 21st Sept. **Why??**
Day Length

Day length between sunrise to sunset

- $S_{\text{max}}$ (day length or maximum number of sunshine hours)

\[ S_{\text{max}} = \frac{2}{15} \times \omega_s = \frac{2}{15} \cos^{-1} \left( -\tan \phi \times \tan \delta \right) \]
Hour Angle

Calculate the hour angle at sunrise and sunset on June 21 and December 21 for a surface inclined at an angle of 10° and facing due south (γ = 0°). The surface is located in Mumbai (19°07’ N, 72°51’ E)
On June 21, $\delta = 23.45^\circ$.

Hour angle at sunrise and sunset, $\omega_{st}$

\[
|\omega_{st}| = \min \left[ \cos^{-1} \left( -\tan \phi \tan \delta \right), \cos^{-1} \left\{ -\tan \left( \phi - \beta \right) \tan \delta \right\} \right]
\]

\[
|\omega_{st}| = \min \left[ \cos^{-1} \left( -\tan 19.2^\circ \tan 23.45^\circ \right), \cos^{-1} \left\{ -\tan \left( 19.12^\circ - 10^\circ \right) \tan 23.45^\circ \right\} \right] = \pm 94.0^\circ
\]

On December 21, $\delta = -23.45^\circ$.

$\omega_{st} = \pm 81.4^\circ$
Estimation of solar radiation
at any given time, any location
Measurement of Solar Radiation, and Estimation of Solar Radiation

- Measurement of solar radiation is expensive and time taking
- Models have been developed to estimate solar radiation at given surface
Estimating solar radiation empirically:

Global radiation

Estimation of monthly average global radiation on horizontal surface

\[
\frac{H_{ga}}{H_{oa}} = a + b \left( \frac{S_a}{S_{max a}} \right)
\]

Where,

- \( H_{ga} \rightarrow \) monthly averaged daily global radiation on a horizontal surface
- \( H_{oa} \rightarrow \) monthly averaged extra-terrestrial solar radiation at horizontal surface (at top of atmosphere)
- \( S_a \) and \( S_{max a} \rightarrow \) monthly averaged daily sunshine hours and maximum possible daily sunshine hours (the day length) at a given location.
- \( a \) and \( b \rightarrow \) constant
Values of constants $a$ and $b$

<table>
<thead>
<tr>
<th>Location</th>
<th>$a$</th>
<th>$b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ahmedabad, India</td>
<td>0.28</td>
<td>0.48</td>
</tr>
<tr>
<td>Atlanta, Georgia, USA</td>
<td>0.38</td>
<td>0.26</td>
</tr>
<tr>
<td>Brownsville, Texas, USA</td>
<td>0.35</td>
<td>0.31</td>
</tr>
<tr>
<td>Buenos Aires, Argentina</td>
<td>0.26</td>
<td>0.50</td>
</tr>
<tr>
<td>Charleston, S. C., USA</td>
<td>0.48</td>
<td>0.09</td>
</tr>
<tr>
<td>Bangalore, India</td>
<td>0.18</td>
<td>0.64</td>
</tr>
<tr>
<td>Hamburg, Germany</td>
<td>0.22</td>
<td>0.57</td>
</tr>
<tr>
<td>Malange, Angola</td>
<td>0.34</td>
<td>0.34</td>
</tr>
<tr>
<td>Miami, Florida, USA</td>
<td>0.42</td>
<td>0.22</td>
</tr>
<tr>
<td>Nagpur, India</td>
<td>0.27</td>
<td>0.50</td>
</tr>
<tr>
<td>New Delhi, India</td>
<td>0.25</td>
<td>0.57</td>
</tr>
<tr>
<td>Nice, France</td>
<td>0.17</td>
<td>0.63</td>
</tr>
<tr>
<td>Pune, India</td>
<td>0.31</td>
<td>0.43</td>
</tr>
<tr>
<td>Rafah, Egypt</td>
<td>0.36</td>
<td>0.35</td>
</tr>
<tr>
<td>Stanleyville, Congo</td>
<td>0.28</td>
<td>0.39</td>
</tr>
<tr>
<td>Tamanrasset, Algeria</td>
<td>0.30</td>
<td>0.43</td>
</tr>
</tbody>
</table>

- The values of the constant is empirically obtained from known data.

Ref: Lof J.A. et al, 1966
Estimation of Extra-terrestrial solar radiation – horizontal surface

$H_{oa}$ is equal to $H_o$ if calculated on following days of month:
January 17, February 16, March 16, April 15, May 15, June 11, July 17, August 16, September 15, October 15, November 14 and December 10.

The monthly averaged daily solar extra-terrestrial radiation

\[
H_o = S \int \cos \theta dt \\
H_o = S \left(1 + 0.033 \cos \frac{360n}{365}\right) \left(\int \left(\sin \phi \sin \delta + \cos \phi \cos \delta \sin \omega\right) dt\right) \\
dt = \frac{180\omega}{15\pi} \\
H_o = \frac{24}{\pi} S \left(1 + 0.033 \cos \frac{360n}{365}\right) \left(\omega_s \sin \phi \sin \delta + \cos \phi \cos \delta \sin \omega_s\right)
\]

$S$ is in W/m² the $H_o$ will be W-Hour/m²
Problem:

Estimate the monthly average daily global radiation on the horizontal surface at Nagpur (21.06N, 79.03E) during month of March if the average sunshine hours per day is 9.2. Assume values for \( a = 0.27 \) and \( b = 0.50 \)

\[
\frac{H_{ga}}{H_{oa}} = a + b \left( \frac{S_a}{S_{\text{max,a}}} \right)
\]

**Given:** \( \Phi, a, b, S_a \)

**Required:** \( \delta, \omega_s, S_{\text{max,a}}, H_{oa}, n, S \)

\[
H_o = H_{oa} = \frac{24}{\pi} S \left( 1 + 0.033 \cos \frac{360n}{365} \right) (\omega_s \sin \phi \sin \delta + \cos \phi \cos \delta \sin \omega_s)
\]

\[
\delta = 23.34 \sin \left( \frac{360}{365} (284 + 75) \right)
\]

\[
\cos \omega_s = - \tan \phi \tan \delta
\]

\[
\omega_s = \cos^{-1} (- \tan \phi \tan \delta)
\]
Solution:

On March 16, n=75
δ= -2.4177

\[ \omega_s = \cos^{-1}(-\tan 21.15 \tan - 2.4177) \]

\[ \omega_s = 89.0640 \]

Day length=11.8752 hr, Sunrise and sunset hours?

\[ H_o = \frac{24}{\pi} \times 1.367 \times 3600 \left(1 + 0.033 \cos \frac{360 \times 57}{365}\right) \left(1.5544 \sin 21.15 \sin - 2.4177 + \cos 21.15 \cos - 2.4177 \sin 89.0640\right) \]

\[ H_o = 34140.2 \text{ kJ/m}^2\text{-day} \]

\[ H_{ga} = 22442.46 \text{ kJ/m}^2\text{-day} \]
Monthly averaged daily Diffuse radiation

\[
\frac{H_{da}}{H_{ga}} = 1.391 - 3.560K_T + 4.189K_T^2 - 2.137K_T^3
\]

\[
\frac{H_{da}}{H_{ga}} = 1.311 - 3.022K_T + 3.427K_T^2 - 1.821K_T^3
\]

\(\omega_s \leq 81.4^\circ\) and \(0.3 \leq K_T \leq 0.8\)

\(\omega_s > 81.4^\circ\) and \(0.3 \leq K_T \leq 0.8\)

\(\omega_s = \) sunrise hour angle

\(K_T = \) sky monthly averaged clearness index, \(= \frac{H_{ga}}{H_{oa}}\)

Typically diffuse radiation is about 10 to 20% of the global radiation on horizontal surface

Ref: Erbs et al., 1982
Monthly averaged daily Diffuse radiation- horizontal surface

Studies on Indian solar radiation data

\[ 0.3 \leq K_T \leq 0.7 \]

\[
\frac{H_{da}}{H_{ga}} = 1.411 - 1.696K_T
\]

One other analysis presented following

\[
\frac{H_{da}}{H_{ga}} = 1.354 - 1.570K_T
\]

• \( K_T = \) sky monthly averaged clearness index, = \( H_{ga} / H_{oa} \)

• Typically diffuse radiation is about 10 to 20% of the global radiation on horizontal surface

• Diffuse radiation component in India is larger than Europe
Radiation on tilted south facing surface

- Angle of sun rays with $\beta=0$
  \[ \cos \theta = \sin \phi \sin \delta + \cos \phi \cos \delta \cos \omega = \cos \theta_z \]

- Angle of sun rays with $\gamma=0$
  \[ \cos \theta = \sin \delta \sin(\phi - \beta) + \cos \delta \cos \omega \cos(\phi - \beta) \]

- **Tilt factor** for beam radiation
  \[ r_b = \frac{\cos \theta}{\cos \theta_z} \]

- **Tilt factor** for diffuse radiation
  \[ r_d = \frac{1 + \cos \beta}{2} \]

- **Tilt factor** for reflected radiation
  \[ r_d = \rho \left( \frac{1 - \cos \beta}{2} \right) \]

* $\rho$ is reflectivity of surrounding, 0.1 to 0.2
Radiation on tilted south facing surfaces

• Total radiation falling on tilted surface at any instant

\[ I_T = I_b r_b + I_d r_d + (I_b + I_d) r_r \]

\[
\frac{I_T}{I_g} = \left(1 - \frac{I_d}{I_g}\right) r_b + \frac{I_d}{I_g} r_d + r_r
\]

• Total daily radiation falling on south tilted surface, \( \gamma = 0 \)

\[
\frac{H_T}{H_g} = \left(1 - \frac{H_d}{H_g}\right) R_b + \frac{H_d}{H_g} R_d + R_r
\]

\[
R_b = \frac{\omega_{st} \sin \delta \sin(\phi - \beta) + \cos \delta \cos \omega_{st} \cos(\phi - \beta)}{\omega_s \sin \delta \sin \phi + \cos \delta \cos \phi \cos \omega_s}
\]

\[
R_d = r_d \quad \& \quad R_r = r_r
\]

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Introduction to Renewable Energy Technologies
Thank you for your attention

Chetan S. Solanky