Lecture 16
Introduction to Controllers and PID Controllers

Process Control
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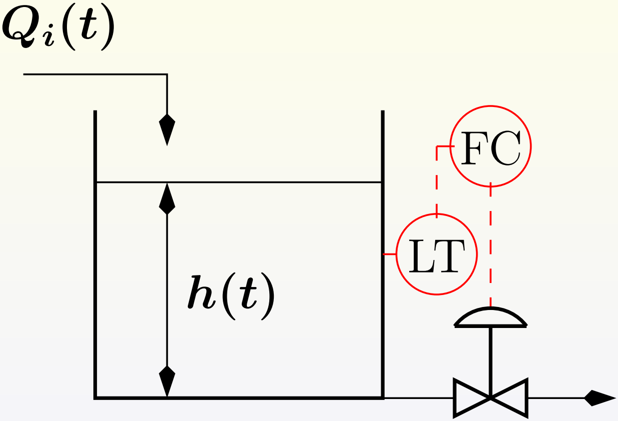
Outline

1. **Recalling control loop components**
2. **Proportional Controller**
3. **Proportional Integral Controller**

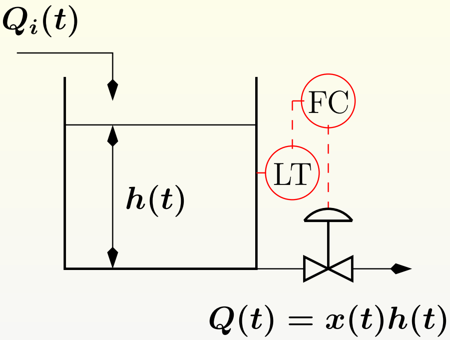
1. Recalling control loop components

Recall: Feedback Control of Flow System



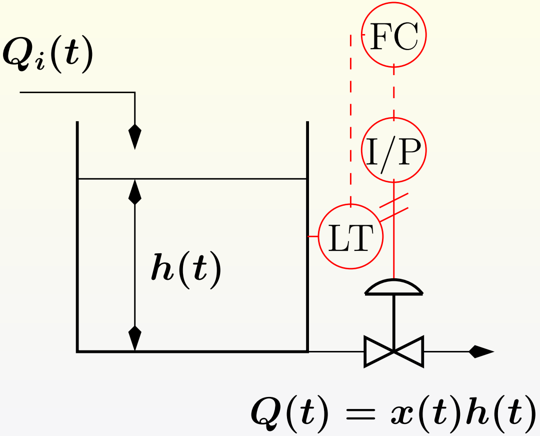
$$Q(t) = x(t)h(t)$$

Control loop components for flow control



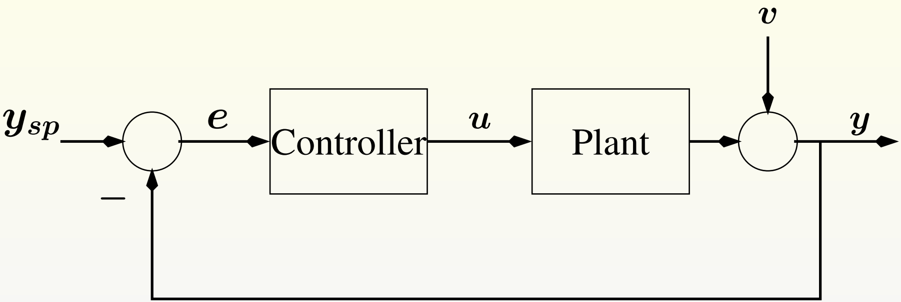
- **Sensor/Transducer: measures level**
- **LT denotes level transmitter, including level sensor**
- **Feedback controller: FC**
- **End control element: control valve**

Refinement of control loop components



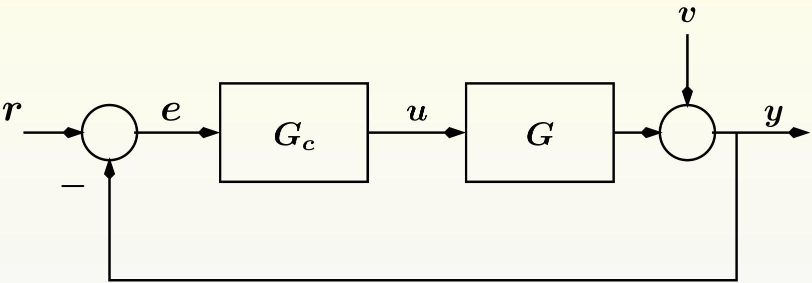
- **Flow control output goes to I/P converter**
- **I/P converter converts current into pressure signal**

Recall: closed loop system



- **We will club the sensor, actuator, etc. dynamics into controller or plant or both**
- **Arrive at the above simplified block diagram**
- **Analysis becomes easy**

Recall: closed loop system



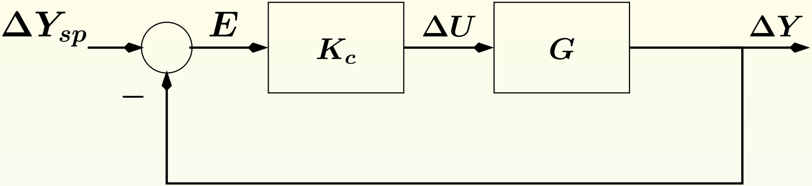
- For G_c, we will substitute different controllers
- We will often use I and II order systems for G
- Often we will ignore noise, i.e. take $v = 0$
- We will interchangeably use r and y_{sp}

2. Proportional Controller

Feedback controllers

- **Will study PID controllers**
- **Will begin with the proportional-controller/proportional-mode**

Proportional Controller



- **I have put Δ to emphasise the fact that I am using deviation variables**
- **Proportional control law: $G_c = K_c$, a constant**
- **The G_c block implements the following:**

$$u(t) = \bar{u} + K_c e(t)$$

where $\bar{u}$ is steady state value or bias and $e(t) \leftrightarrow E(s)$

Proportional Controller

Proportional control law:

$$u(t) = \bar{u} + K_c e(t)$$

where $\bar{u}$ is steady state value or bias.

$$\Delta u(t) = u(t) - \bar{u} = K_c e(t)$$

In the textbook, p(t) is manipulated variable:

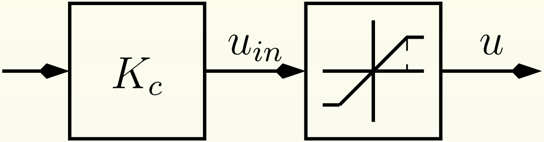
$$p(t) = \bar{p} + K_c e(t)$$

Alternately, use Proportional Band (PB):

$$PB = \frac{100}{K_c}\%$$

Linear control law is linear in a range

Control law saturates beyond a value:



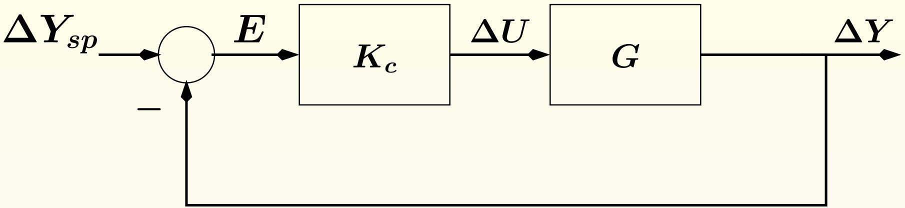
We assume to work in linear range:

$$u(t) = \bar{u} + K_c e(t)$$

Write it as

- $u(t) - \bar{u}(t) = K_c e(t)$
- Rewrite it as $\Delta u(t) = K_c e(t)$
- Take Laplace transform: $\Delta U(s) = K_c E(s)$
- Gain of the controller $= K_c$

Proportional controller in a feedback loop



What is the closed loop relationship?

$$\Delta Y(s) = \frac{K_c G(s)}{1 + K_c G(s)} \Delta Y_{sp}(s)$$

Let G_{cl} denote closed loop transfer function:

$$\Delta Y(s) = G_{cl} \Delta Y_{sp}(s)$$

What does response to a step in Y_{sp} mean?

Step response of proportional controller

- **What does a step change in $y_{sp}(t)$ mean?**

$$\Delta Y(s) = \frac{K_c G(s)}{1 + K_c G(s)} \Delta Y_{sp}(s)$$

- **Calculate $\Delta y(t = \infty)$ for $\Delta Y_{sp} = 1/s$:**

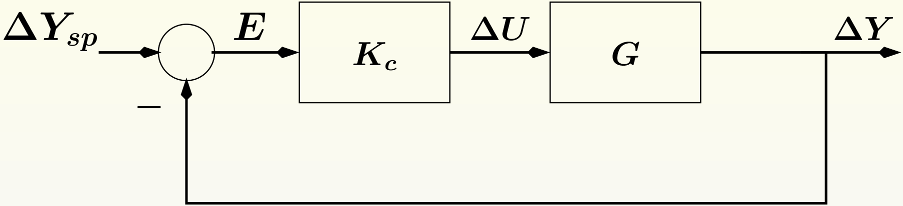
$$\Delta y(t = \infty) = \frac{K_c G(0)}{1 + K_c G(0)}$$

If $G(s) = K/(\tau s + 1)$,

$$\Delta y(t = \infty) = \frac{K_c K}{1 + K_c K}$$

What is the steady state offset (or error)?

Steady State Offset



The steady state offset (or error) is

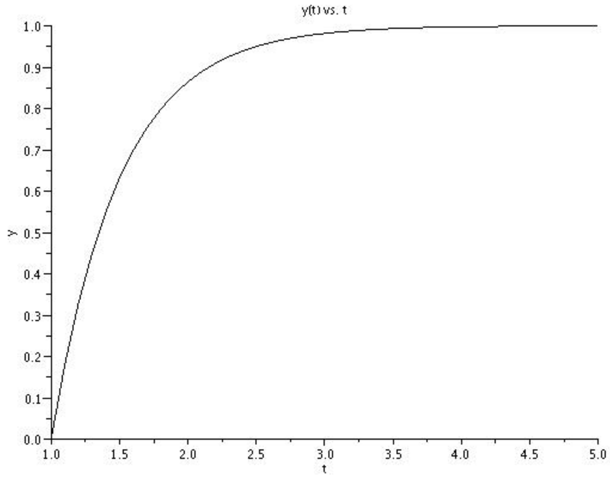
1. not known, not enough information is given
2. KcK/(1+KcK)
3. 1-KcK/(1+KcK)

Answer: 3

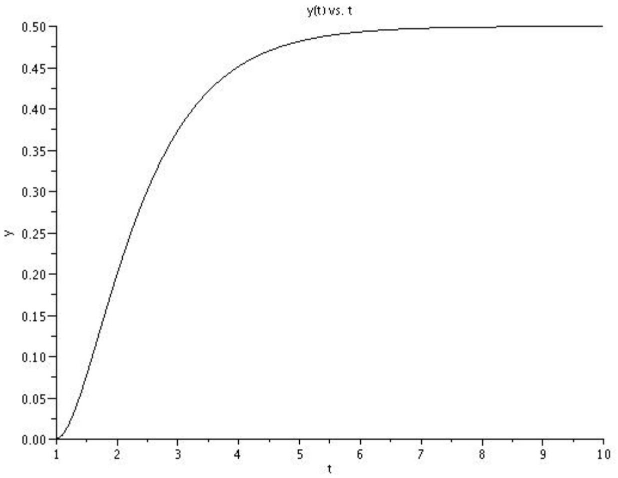
Can we zero the offset?

- $\Delta y(t = \infty) = \dfrac{K_c K}{1 + K_c K}$
- Steady state error $= 1 - \dfrac{K_c K}{1 + K_c K} = \dfrac{1}{1 + K_c K}$
- Why subtract from 1?
- Because **unit** step change was used
- Can we make the steady state error **zero?**
- Can make it as small as required by **increasing** K_c
- Are there any undesirable side effects?
- Because of unmodelled dynamics, G may actually be a second order system!
- **Recall the step response of SBHS!**

Step response of $\mathbf{G(s)} = \frac{1}{0.5s+1}$



Step response of $\mathbf{G(s)} = \dfrac{1}{(s+1)(s+2)}$



MCQ: Increasing K_c

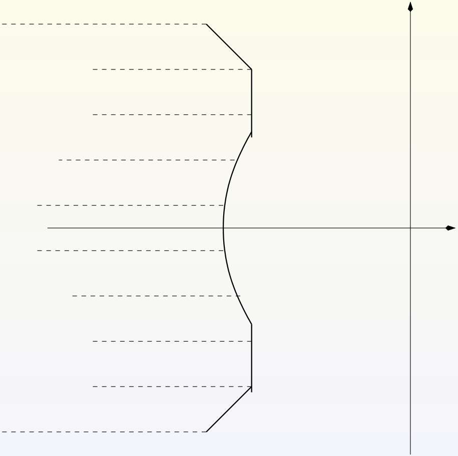
By increasing K_c indefinitely in a closed loop system with an actual plant,

1. **The system will respond better and better with the steady state offset decreasing**
2. **The system is likely to oscillate, although, the steady state error may decrease**
3. **Cannot say, as first order systems will not oscillate**

Answer: 2

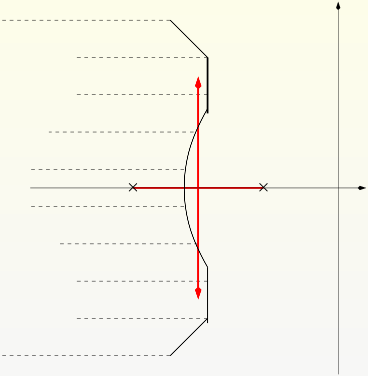
Side effects of increasing K_c indefinitely

Recall the desired region:



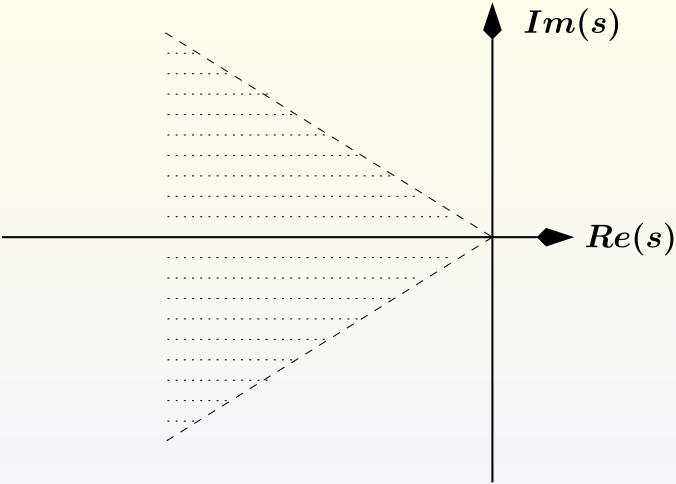
Is there any problem in increasing K_c indefinitely?

Shortcomings of indefinite increase in K_c



- **Increase K_c to bring the closed loop poles inside desired region**
- **Indefinite increase of K_c will take root locus outside desired region**
- **What condition is violated?**

Recall small overshoot condition



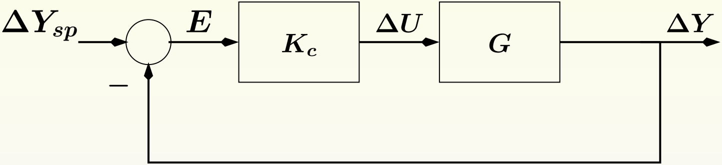
- **If poles are outside shaded region,**
- **Large overshoots and hence large oscillations**

Trade off in increasing K_c

- Recall steady state error:
- Steady state error $= \dfrac{1}{1 + K_c K}$
- Can decrease error with large K_c
- Unfortunately, this may result in unacceptable oscillations
- How do we handle this situation?

Trade off between offset & control action

Recall the closed loop system:



- Zero error $E \Rightarrow \Delta U = 0$
- This implies zero control action with respect to nominal value of U
- Servo/tracking control (set point changes) cannot be implemented
- Can we have both $E = 0$ and $\Delta U \neq 0$ with finite K_c?

Introducing Integral Mode

$$u(t) = \bar{u} + \frac{1}{\tau_i}\int_0^t e(w)dw$$

- Even a 0 value of e(t) can give rise to nonzero values of $\Delta u(t)$!
- τ_i is reset time or integral time
- Recall $u(t) \leftrightarrow U(s)$
- $$\Delta U(s) = \frac{1}{\tau_i s}E(s)$$
- Normally, we use a proportional-integral (PI) controller

PI controller

PI controller:

$$u(t) = \bar{u} + K_c \left[e(t) + \frac{1}{\tau_i} \int_0^t e(w)dw \right]$$

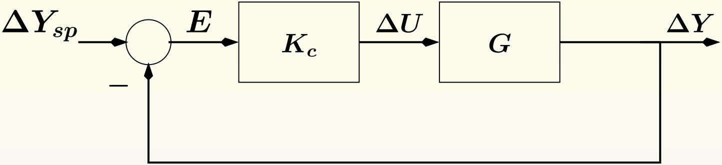
or

$$\Delta U(s) = K_c \left[1 + \frac{1}{\tau_i s} \right] E(s)$$

- **What is the steady state offset now?**

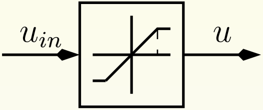
Offset of I order system with PI Controller

Recall the closed loop system:



- Take G to be $\dfrac{K}{\tau s + 1}$
- In the place of K_c, use a PI controller,
- i.e. replace K_c with $G_c = K_c \left[1 + \dfrac{1}{\tau_i s}\right]$
- Calculate the step response of the C. L. System
- Calculate $\lim_{t\to\infty} \Delta y(t)$

Why not use integral mode ALONE?



- Problem due saturation of control effort
- When required control effort cannot be created, there will be offset
- No option but to live with this error
- Integral mode will crank up the control action, thinking that a larger control effort is required
- Will take time to unwind this controller

Why not use integral mode alone? - ctd

- **One way to handle this: monitor for mismatch and disable the integral mode**
- **For another method, see Digital Control by Moudgalya, John Wiley & Sons, 2007**
- **Known as integral windup**

Integral mode generalises proportional mode

PI controller:

$$\Delta U(s) = K_c \left[1 + \frac{1}{\tau_i s}\right] E(s)$$

or

$$\Delta u(t) = K_c \left[e(t) + \frac{1}{\tau_i}\int_0^t e(w)dw\right]$$

$$= K_c e(t) + K_c \frac{1}{\tau_i}\int_0^t e(w)dw$$

With e constant, after every $t = \tau_i$, $K_c e$ gets added!

Characteristics of Integral Mode

- **Used to remove steady state offset**
- **For open loop stable plants,**
- **increase in integral action generally results in**
- **decreased steady state offset and**
- **increased oscillations**
- **Remember this while tuning integral mode**

What we learnt today

- **Proportional Controller**
- **Proportional Integral Controller**

Thank you