Lecture 18
PID: Ziegler Nichols Tuning, Direct synthesis

Process Control
Prof. Kannan M. Moudgalya

IIT Bombay
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1. Ziegler Nichols Tuning
2. Direct synthesis
3. Calculation of PID by Direct Synthesis
PID controller has proportional, integral and derivative modes:

\[ u(t) = \bar{u} + K_c \left[ e(t) + \frac{1}{\tau_i} \int_0^t e(w)dw + \tau_d \frac{de(t)}{dt} \right] \]

- \( e \) is error, \( u \) is control effort and \( \bar{u} \) is the steady state (or bias) value of \( u \)
- It has three tuning parameters, \( K_c, \tau_i, \tau_d \)
- Recall the tuning guidelines to change these parameters
Summary of PID Controller

Proportional mode:
- Most popular control mode. Increase in proportional mode generally results in
  - Decreased steady state offset
  - Increased oscillations

Integral Mode:
- Used to remove steady state offset. Increase in integral mode generally results in
  - Zero steady state offset
  - Increased oscillations
Summary of PID Controller

Derivative Mode:

- Mainly used for prediction purposes
- Increase in derivative mode results in
  - Decreased oscillations and improved stability
  - Sensitive to noise
- Used extensively by EE/ME
- Less used by chemical engineers, mainly because of noise problems

PID is most popular in industry
1. Ziegler Nichols Tuning
Applicable only to stable systems

Give a unit step input and get

\[ R = \frac{K}{\tau} \]

1. the time lag after which the system starts responding (L),
2. the steady state gain (K) and
3. the time the output takes to reach the steady state, after it starts responding (\(\tau\))
Reaction Curve Method Ctd.

\[ R = \frac{K}{\tau} \]

PID settings are given below:

<table>
<thead>
<tr>
<th></th>
<th>( K_c )</th>
<th>( \tau_i )</th>
<th>( \tau_d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>( \frac{1}{RL} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PI</td>
<td>( \frac{0.9}{RL} )</td>
<td>3L</td>
<td>2L</td>
</tr>
<tr>
<td>PID</td>
<td>( \frac{1.2}{RL} )</td>
<td>2L</td>
<td>0.5L</td>
</tr>
</tbody>
</table>

Consistent units should be used
Stability Method - Ziegler Nichols Tuning

Another method to tune PID parameters:

- Close loop with a proportional controller
- Gain of controller is increased until the closed loop system becomes unstable
- At the verge of instability, note down controller gain $K_u$, period of oscillation $P_u$
- PID settings are given below:

<table>
<thead>
<tr>
<th>Mode</th>
<th>$K_c$</th>
<th>$\tau_i$</th>
<th>$\tau_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>$0.5K_u$</td>
<td>$P_u/1.2$</td>
<td>$P_u/8$</td>
</tr>
<tr>
<td>PI</td>
<td>$0.45K_u$</td>
<td>$P_u/2$</td>
<td></td>
</tr>
<tr>
<td>PID</td>
<td>$0.6K_u$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Consistent units should be used
2. Direct Synthesis
Motivation for direct synthesis

- We get PID control parameters by trial and error
- We need a starting point
- Tuning methods give this starting point
- Ziegler Nichols tuning is one approach
- We need a set of different tuning methods - no single method will work for every plant
- Direct synthesis is another tuning method
What is direct synthesis?

- Specify the desired closed loop performance
- Back calculate the controller
Direct synthesis

- $G_c$ to be decided
- Let $r$ be $y_{sp}$
- Closed loop relation is

$$y = \frac{G_c G}{1 + G_c G} y_{sp}$$
Direct synthesis - ctd

- Closed loop relation is

\[
y = \frac{G_c G}{1 + G_c G} y_{sp}
\]

- We obtain

\[
\frac{y}{y_{sp}} = \frac{G_c G}{1 + G_c G}
\]

- If \( \left( \frac{y}{y_{sp}} \right) \) is given, we can calculate \( G_c \), because we know \( G \)
Calculation of $G_c$ by direct synthesis

- Start with $\alpha \triangleq \frac{y}{y_{sp}} = \frac{G_c G}{1 + G_c G}$
- Calculate $G_c$ in terms of $\alpha$
- Reciprocal: $\frac{1}{1 + G_c G} = \frac{1}{(y/y_{sp})}$
- Simplify: $1 + \frac{1}{G_c G} = \frac{(y/y_{sp})}{1 - (y/y_{sp})}$
- We arrive at the required relation:

$$G_c = \frac{1}{G} \frac{(y/y_{sp})}{1 - (y/y_{sp})}$$
An open loop system is slow. We want to make it faster. In closed loop,

1. The time constant should be smaller
2. The time constant should be larger
3. The gain should be larger
4. The gain should be smaller

Ans: 1
Specification of response

- Start with \( G_c = \frac{1}{G} \frac{(y/y_{sp})}{1 - (y/y_{sp})} \)

- Suppose \( G \) is first order with no time delay

- i.e. \( G = \frac{K}{\tau s + 1} \)

- Explore demanding \( \frac{y}{y_{sp}} = \frac{1}{\tau_{cl}s + 1} \)

- What should be the desired \( (y/y_{sp}) \)?

- Ch.E. systems are generally sluggish

- Want the controller to make it faster

- How should \( \tau_{cl} \) and \( \tau \) be related? \( \tau_{cl} < \tau \)

- One half, one third, etc.
3. Calculation of PID by Direct Synthesis
Calculation of DS controller

- Controller is \( G_c = \frac{1}{G} \frac{(y/y_{sp})}{1 - (y/y_{sp})} \), \( G = \frac{K}{(\tau s + 1)} \)

- We have chosen \( \frac{y}{y_{sp}} = 1 \) \( \frac{y_{sp}}{\tau_{cl}s + 1} \)

- Substitute and simplify and obtain \( G_c \)

- We obtain \( G_c = \frac{\tau s + 1}{K} \frac{1}{\tau_{cl}s} \)

- Simplifying it, \( G_c = \frac{\tau}{K\tau_{cl}} \left[ 1 + \frac{1}{\tau s} \right] \)

- This is of the form, \( G_c = K_c \left[ 1 + \frac{1}{\tau_i s} \right] \)

- with \( K_c = \frac{\tau}{K\tau_{cl}} \), \( \tau_i = \tau \), a PI controller!
Plant with time delay

- Want to design controller for \( G = \frac{K}{\tau s + 1} e^{-\theta s} \)

- Demand this response: \( \frac{y}{y_{sp}} = \frac{e^{-Ds}}{\tau_{cl} s + 1} \)

- \( D \) and \( \tau_{cl} \) are to be decided

- Controller formula: \( G_c = \frac{1}{G} \frac{1 - (y/y_{sp})}{1 - (y/y_{sp})} \)

- Substituting, \( G_c = \frac{\tau s + 1}{K} \frac{e^{-(D - \theta)s}}{\tau_{cl} s + 1 - e^{-Ds}} \)

- \( D < \theta \Rightarrow \) prediction, need \( D \geq \theta \), Choose \( D = \theta \)
Closed loop time delay should be at least equal to open loop time delay, because, we want

1. the closed loop system to be fast
2. the closed loop system to be stable
3. the controller to be realisable
4. the controller to be stable

Ans: 3
From the previous slide,

\[ G_c = \frac{\tau s + 1}{K} \frac{e^{-(D-\theta)s}}{\tau_{cl}s + 1 - e^{-Ds}} \]

- \( D = \theta \)
- Use a first order approximation for \( e^{-Ds} \)

\[ G_c = \frac{\tau s + 1}{K} \frac{1}{(\tau_{cl} + \theta)s} \]

- What controller is this? Recall the expression in blue. PI controller!
Second order plant with time delay

- **Plant is** \( G = \frac{Ke^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)} \)

- **Desired response is** \( \frac{y}{y_{sp}} = \frac{e^{-D s}}{\tau_{cl} s + 1} \)

- **Derive it now**

- \( G_c = \frac{1}{G} \left( \frac{y}{y_{sp}} \right) \)

- \( G_c = \frac{(\tau_1 s + 1)(\tau_2 s + 1)}{Ke^{-\theta s}} \cdot \frac{\frac{e^{-D s}}{\tau_{cl} s + 1}}{1 - \frac{e^{-D s}}{\tau_{cl} s + 1}} \)
Derivation for 2nd Order Plant with Delay

\[ G_c = \frac{(\tau_1 s + 1)(\tau_2 s + 1)}{Ke^{-\theta s}} \cdot \frac{e^{-Ds}}{\tau_{cl} s + 1} \cdot \frac{\tau_{cl} s + 1}{1 - e^{-Ds}} \]

\[ = \frac{(\tau_1 s + 1)(\tau_2 s + 1)}{K} \cdot \frac{1}{(\tau_{cl} + \theta)s} \]

\[ = 1 + (\tau_1 + \tau_2)s + \tau_1 \tau_2 s^2 \]

\[ = \frac{\tau_1 + \tau_2}{K(\tau_{cl} + \theta)} \left[ 1 + \frac{1}{(\tau_1 + \tau_2)s} + \frac{\tau_1 \tau_2}{\tau_1 + \tau_2} s \right] \]
Second order plant with time delay

\[ G_c = \frac{\tau_1 + \tau_2}{K(\tau_{cl} + \theta)} \left[ 1 + \frac{1}{(\tau_1 + \tau_2)s} + \frac{\tau_1 \tau_2}{\tau_1 + \tau_2} s \right] \]

- Controller is PID

Comparing coefficients,

\[ K_c = \frac{\tau_1 + \tau_2}{K(\tau_{cl} + \theta)}, \quad \tau_i = \tau_1 + \tau_2, \quad \tau_d = \frac{\tau_1 \tau_2}{\tau_1 + \tau_2} \]
What we learnt today

- ZN Tuning
- Motivation for direct synthesis
- Direct synthesis of
  - First order systems with and without delay
  - Second order systems with and without delay
- Specifications for direct synthesis
Practical issues in PID implementation
Thank you