Lecture 24
Examples of Bode Plots

Process Control
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Thursday, 26 September 2013

Outline

Recall: First order transfer function

- $G(s) = \dfrac{1}{\tau s + 1}$, $G(j\omega) = \dfrac{1}{j\omega\tau + 1}$
- $|G(j\omega)| = \dfrac{1}{\sqrt{\omega^2\tau^2 + 1}}$
- $\omega \ll 1$, $|G(j\omega)| = 1$,

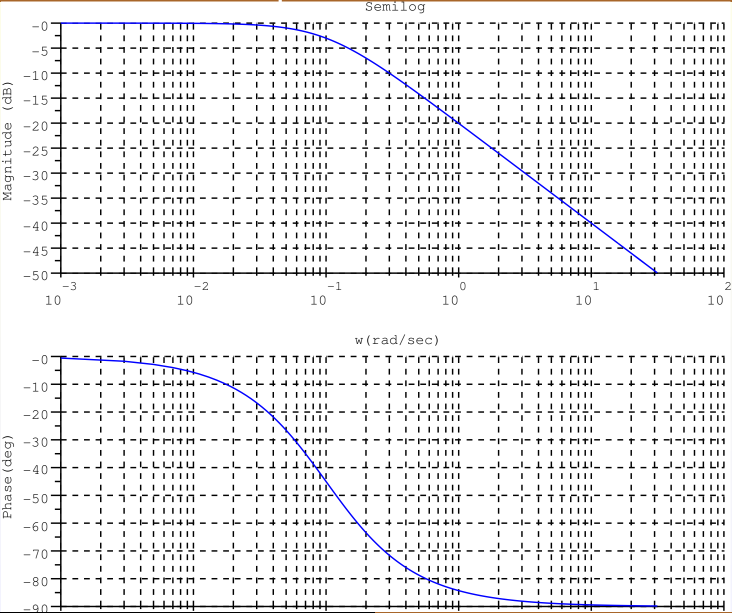
Recall: First order transfer function

- $G(s) = \dfrac{1}{\tau s + 1}$, $G(j\omega) = \dfrac{1}{j\omega\tau + 1}$
- $|G(j\omega)| = \dfrac{1}{\sqrt{\omega^2\tau^2 + 1}}$
- $\omega \ll 1$, $|G(j\omega)| = 1$, $M = 20\log|G(jw)| = 0$
- Asymptote is $M = 0$
- $\omega \gg 1$, $|G(j\omega)| = \dfrac{1}{\omega\tau}$, $M = -20\log\omega\tau$
- Asymptote is $M = -20\log\omega\tau$
- $\omega = \omega_1 \Rightarrow M = -20\log\omega_1\tau$
- $\omega = 10\omega_1 \Rightarrow M = -20\log\omega_1\tau - 20$
- Slope of -20 dB per decade

Corner Frequency

- $G(j\omega) = \dfrac{1}{j\omega\tau + 1}$
- $|G(j\omega)| = \dfrac{1}{\sqrt{\omega^2\tau^2 + 1}}$
- For $\omega \ll 1$, the asymptote is $|G(j\omega)| = 1$
- $\omega \gg 1$, the asymptote is $|G(j\omega)| = \dfrac{1}{\omega\tau}$
- Two asymptotes intersect at $\omega = 1/\tau$
- $w = 1/\tau$ is known as the corner frequency

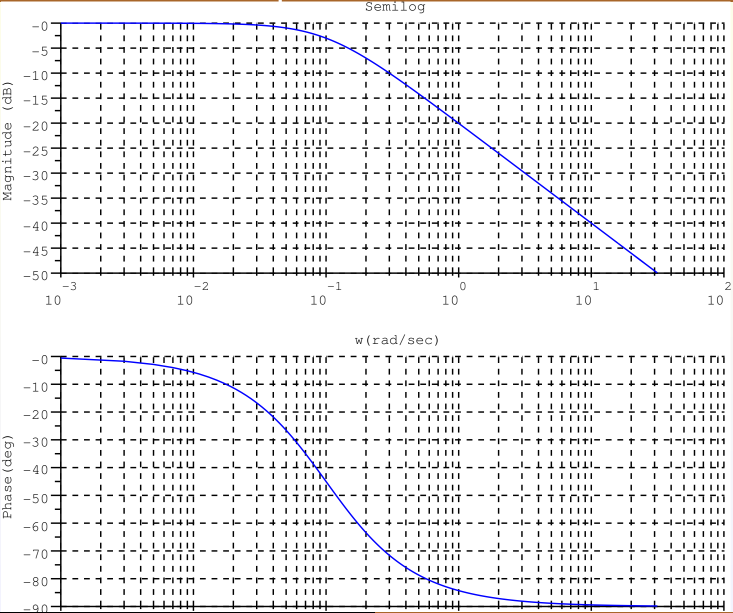
Bode plot of $\frac{1}{10s+1}$ in semilog scale



Value at the corner frequency

- $|G(j\omega)| = \dfrac{1}{\sqrt{\omega^2\tau^2 + 1}}$
- $\omega = 1/\tau$ is known as the corner frequency
- At $\omega = 1/\tau$, what is M?
- $M = -20 \log \sqrt{2} = -10 \log 2 \simeq -3$ dB

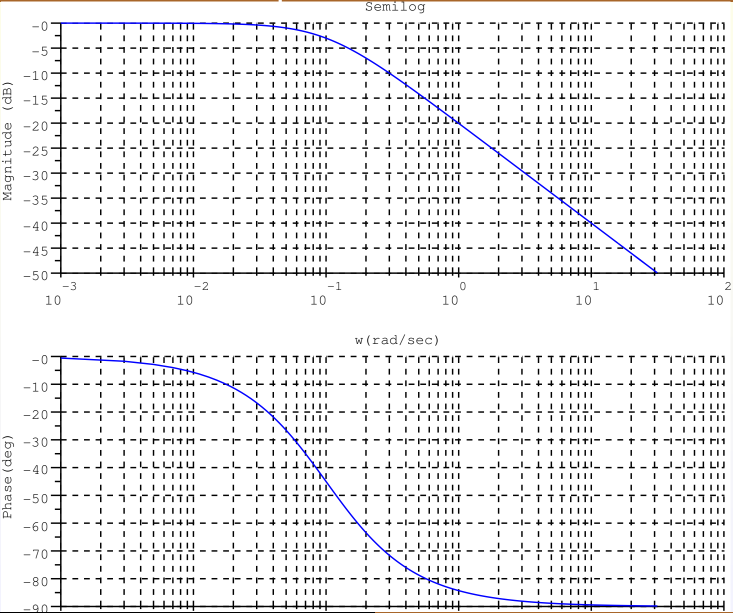
Bode plot of $\frac{1}{10s+1}$ in semilog scale



Phase relations for a simple pole

- $G(s) = \dfrac{1}{\tau s + 1}$, $G(j\omega) = \dfrac{1}{j\omega\tau + 1}$
- $\omega \ll 1$, $G(j\omega) = 1$, $\phi = \angle G(jw) = 0$
- $\omega \gg 1$, $G(j\omega) = \dfrac{1}{j\omega\tau}$, $\phi = -90^\circ$
- For $\omega = 1/\tau$, $G(j\omega) = \dfrac{1}{j1 + 1}$
- $\phi = -45^\circ$

Bode plot of $\frac{1}{10s+1}$ in semilog scale



MCQ: First order system

Bode plot of a first order system has the following properties:

- A **Slope = -20dB/decade for large frequency**
- B **$w = 1/\tau$ at corner frequency**
- C **$\phi = -45^\circ$ at corner frequency**
- D **Phase reached at large frequencies $= -90^\circ$**

Choose the correct answer:

1. **A and B only**
2. **A and C only**
3. **A, B and C only**
4. **All four are correct**

Answer: 4

2. Gain, integral and derivative

Effect of Gain on Magnitude Bode Plot

- $G(s) \triangleq 100G_1(s)$
- $M = 20\log|G(j\omega)|$ and $M_1 = 20\log|G_1(j\omega)|$
- Both M and M_1 are plotted in the same graph, in dB (decibel units)

M and M_1 are related in the following way:

1. M is higher than M_1 by 100 units
2. M is higher than M_1 by 40 units
3. M is lower than M_1 by 100 units
4. The slopes of M and M_1 are different by 100 units

Answer: 2

Effect of Gain on Phase Bode Plot

- $G(s) \triangleq 100G_1(s)$
- $\phi = \angle G(j\omega)$ and $\phi_1 = \angle G_1(j\omega)$
- Both ϕ and ϕ_1 are plotted in the same graph

ϕ and ϕ_1 are related in the following way:

1. ϕ is higher than ϕ_1 by 100 units
2. ϕ is higher than ϕ_1 by 40 units
3. Both ϕ and ϕ_1 plots are identical
4. There is no relation between ϕ and ϕ_1

Answer: 3

Effect of gain

- $G(s) \triangleq KG_1(s)$, $K > 0$
- $M = 20 \log |G(j\omega)| = 20 \log |KG_1(j\omega)|$
- $M = 20 \log K + 20 \log |G_1(j\omega)|$, $K > 0$
- Example: $K = 100$
- $M = 40 + 20 \log |G_1(j\omega)|$
- At every frequency, add 40 dB!
- Phase plots of G_1 and G are identical

Effect of integral mode or pole at zero

- $G(s) = \dfrac{1}{s}$
- $G(j\omega) = \dfrac{1}{j\omega}$
- $M = 20 \log |G(j\omega)| = -20 \log \omega$
- Has a slope of -20 dB per decade
- $\phi = \angle G(j\omega) = -90^{\circ}$

Scilab code bode-5.sce

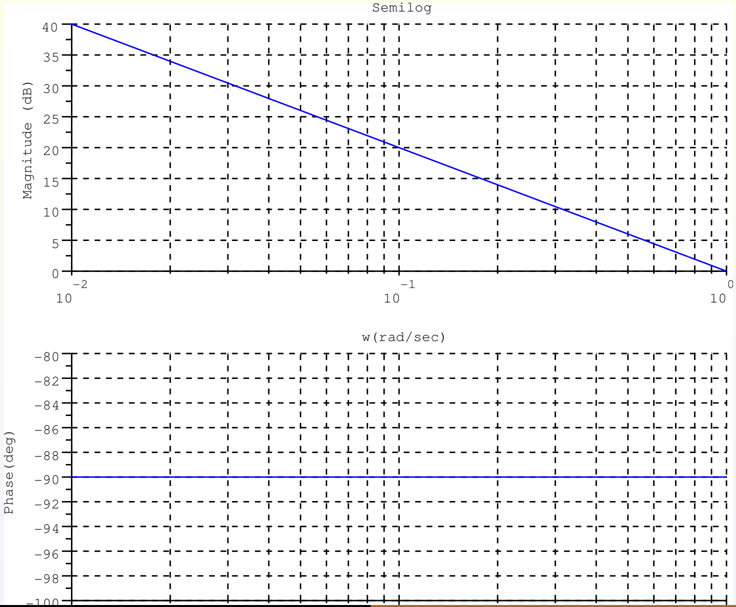
```
exec('bodegen-1.sci');

s = %s;
num = 1;
den = s;

w = 0.01:0.002:%pi^0;
LF = "semilog"

bodegen(num,den,w,LF);
```

Bode plot of a pole at zero



Bode plot of pure derivative action

- $G(s) = s$
- $G(j\omega) = j\omega$
- $M = 20\log|G(j\omega)| = 20\log\omega$
- Has a slope of +20 dB per decade
- $\phi = \angle G(j\omega) = +90^\circ$

Scilab code bode-5.sce

Exchange the values of num and den and execute

3. Adding Bode Plots

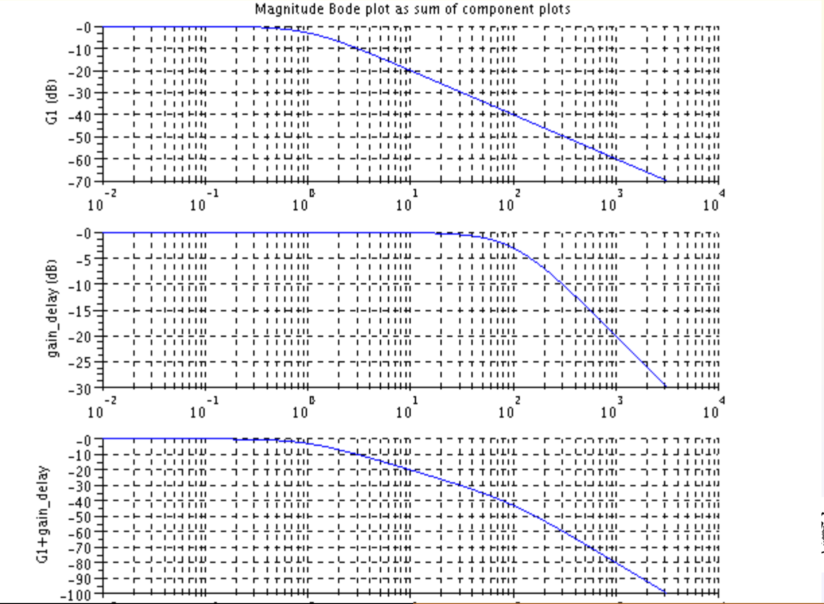
3a. Two first order systems in series

Product of two first order systems

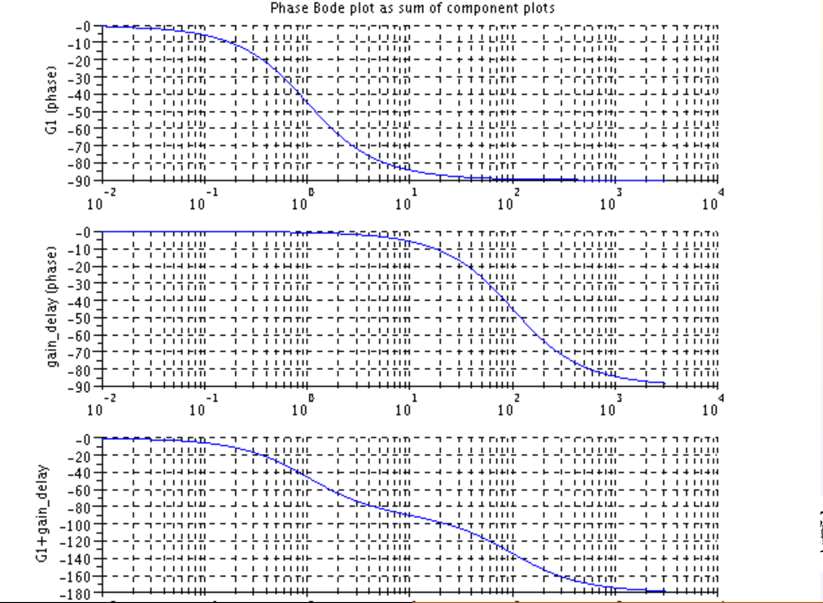
$$G(s) = \frac{1}{s+1}\frac{1}{0.01s+1}$$

- Plot M for each transfer function separately
- What are the corner frequencies? For the first,
- it is 1
- For the second, it is $1/0.01 = 100$
- Add the two
- Draw ϕ for each transfer function separately
- Add the two
- Scilab code and the plots are given next

Magnitude Bode Plot



Phase Bode Plot



Scilab code bode-2.sce

Scilab code:

```
exec('bodesum-2.sci');
s = %s;
G1 = 1/(s+1);
gain = 1/(0.01*s+1);
delay = 0;
w = 0.01:0.008*%pi:1000*%pi;
bodesum_1(G1,delay,gain,w);
```

Scilab code:

```
// Bode plot as a sum of components

function bodesum_1(G1,delay,gain,w)

G1_freq = horner(G1,%i*w);
G1_mag = 20*log10(abs(G1_freq));
gain_freq = horner(gain,%i*w);
gain_mag = 20*log10(abs(gain_freq));

xset('window',0); clf();
subplot(3,1,1)
```

Scilab code bodesum-2.sci II

```
plot2d(w,G1_mag,logflag='ln',style = 2);
xgrid();
xtitle('Magnitude Bode plot as sum of component plots','','G1 (dB)');
subplot(3,1,2)
plot2d(w,gain_mag,logflag="ln",style = 2);
xgrid();
xtitle('','','gain_delay (dB)');
subplot(3,1,3)
```

Scilab code bodesum-2.sci III

```
plot2d(w,G1_mag+gain_mag,logflag="ln
  ",style = 2);
xgrid();
xtitle('','Phase(deg)','G1+
  gain_delay');

G1_ph = phasemag(G1_freq);
gain_ph = phasemag(gain_freq) -delay
  *w*180/%pi;

xset('window',1); clf();
subplot(3,1,1)
```

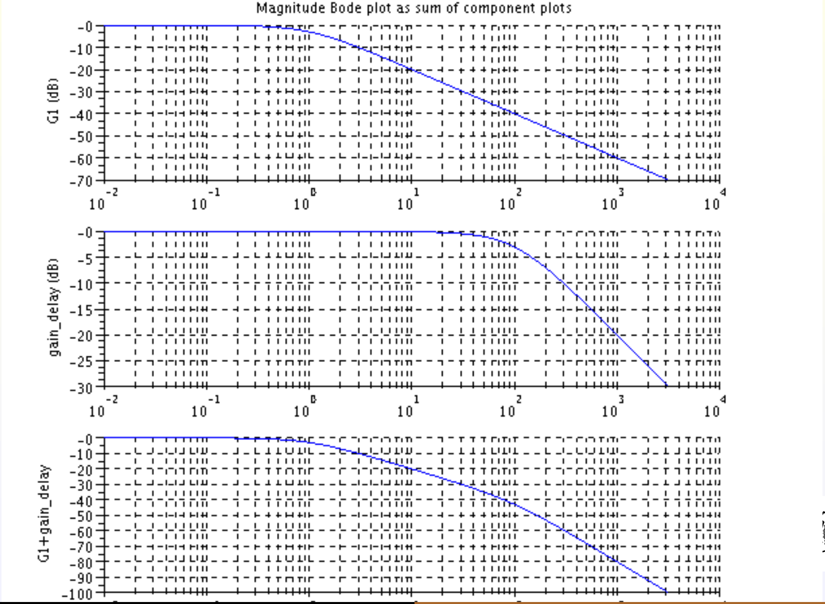
Scilab code bodesum-2.sci IV

```
plot2d(w,G1_ph,logflag='ln',style = 2);
xgrid();
xtitle('Phase␣Bode␣plot␣as␣sum␣of␣component␣plots','','G1␣(phase)');
subplot(3,1,2)
plot2d(w,gain_ph,logflag="ln",style = 2);
xgrid();
xtitle('','','gain_delay␣(phase)');
subplot(3,1,3)
```

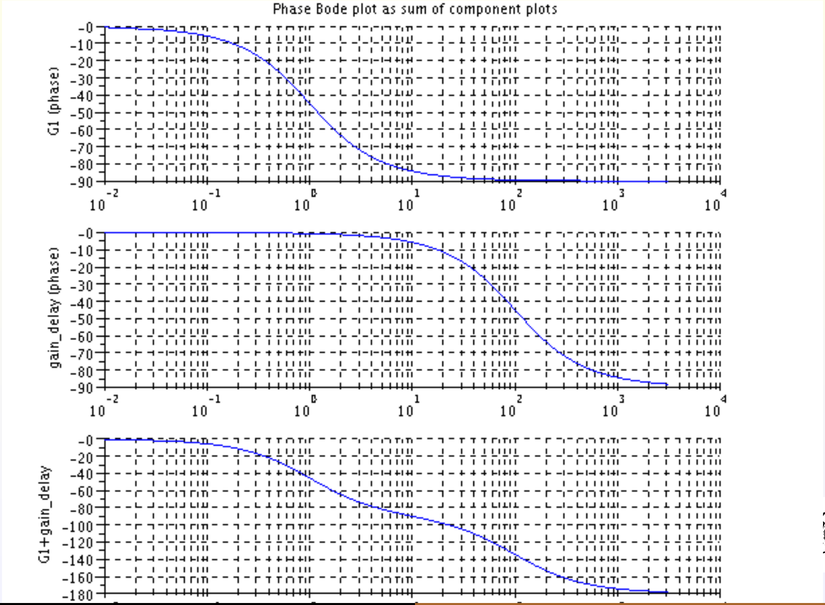
Scilab code bodesum-2.sci V

```
plot2d(w,G1_ph+gain_ph,logflag="ln",
   style = 2);
xgrid();
xtitle('','Phase(deg)','G1+
   gain_delay');
endfunction;
```

Magnitude Bode Plot



Phase Bode Plot



3b. Lead Transfer Function

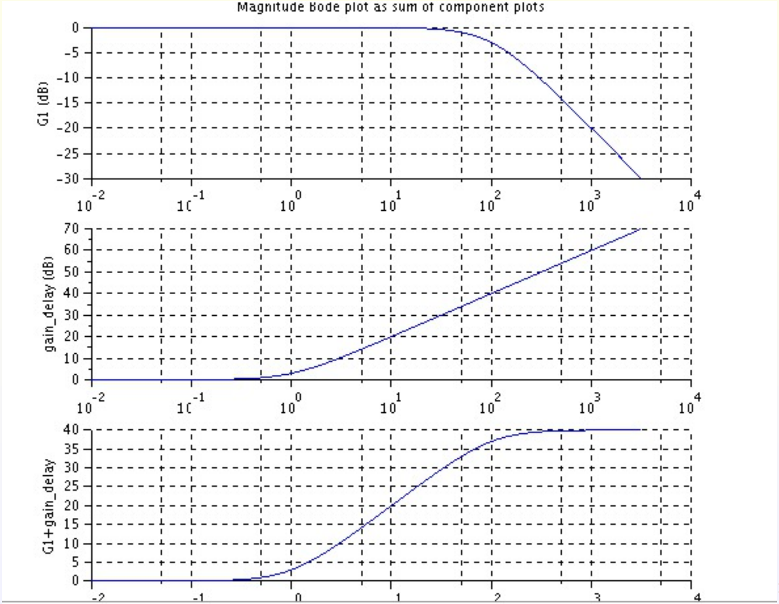
Lead Transfer Function

- Consider the lead transfer function:

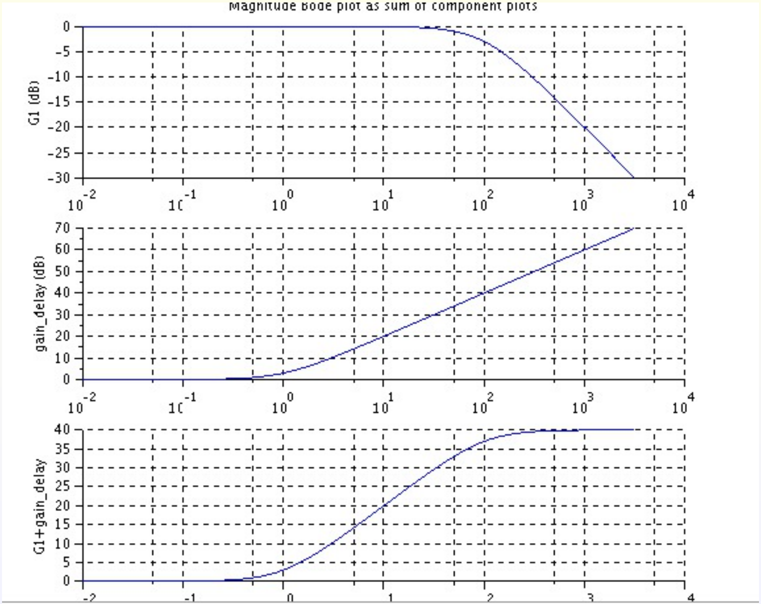
$$G(s) = \frac{s+1}{0.01s+1}$$

- Corner frequencies are 1 and 100
- Magnitude plot of $s+1$ has a slope of +20 dB
- Phase plot of $s+1$ increases, goes to 90°
- Magnitude plot of $1/(0.01s+1)$ has a slope of -20 dB
- Phase plot of $1/(0.01s+1)$ decreases, goes to -90°
- Add the two
- Scilab code is given next

Magnitude Bode Plot



Phase Bode Plot



Scilab code bode-2a.sce

```
exec('bodesum-2.sci');
s = %s;
G1 = 1/(0.01*s+1);
gain = (s+1);
delay = 0;
w = 0.01:0.008*%pi:1000*%pi;
bodesum_1(G1,delay,gain,w);
```

3c. First order system with delay

Effect of Delay on Magnitude Bode Plot

- $G(s) \triangleq G_1(s)e^{-Ds}$
- $M = 20\log|G(j\omega)|$ and $M_1 = 20\log|G_1(j\omega)|$
- Both M and M_1 are plotted in the same graph, in dB (decibel units)

M and M_1 are related in the following way:

1. M is lower than M_1 by D units
2. M is lower than M_1 by 1 unit
3. M and M_1 are identical
4. There is no relation between M and M_1

Answer: 3

Effect of Delay on Phase Bode Plot

- $G(s) \triangleq G_1(s)e^{-Ds}$
- $\phi = \angle G(j\omega)$ and $\phi_1 = \angle G_1(j\omega)$
- Both ϕ and ϕ_1 are plotted in the same graph

ϕ and ϕ_1 are related in the following way:

1. ϕ is lower than ϕ_1 by D units
2. ϕ is obtained from ϕ_1 by subtracting $D\omega$ at every ω
3. Both ϕ and ϕ_1 plots are identical
4. There is no relation between ϕ and ϕ_1

Answer: 2

Scilab code for delay

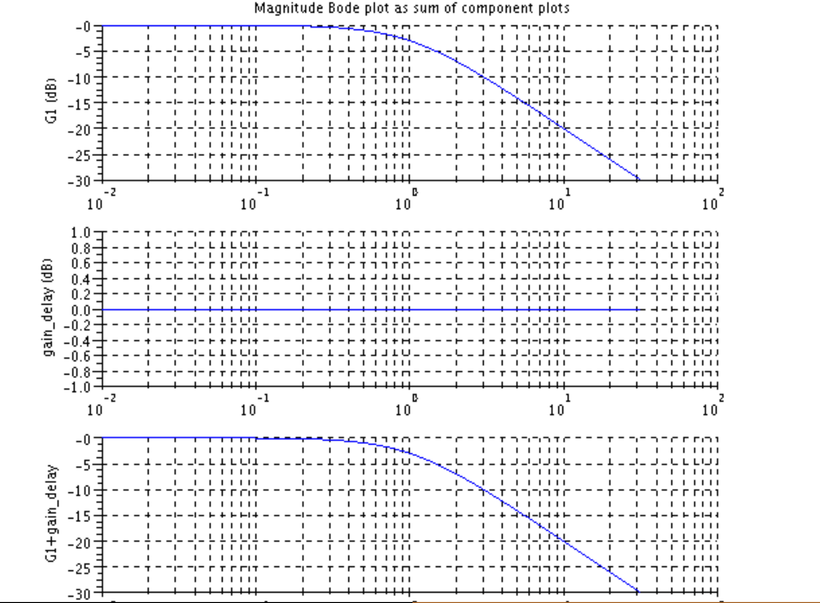
- $G(s) = e^{-Ds}$
- $G(j\omega) = e^{-jD\omega}$
- $G(j\omega) = \cos D\omega - j \sin D\omega$
- $\phi = \angle G(j\omega) = \tan^{-1}\left[-\dfrac{\sin D\omega}{\cos D\omega}\right]$
- $\phi = -D\omega$
- What about magnitude plot?
- $M = 1$ for all ω

Scilab code bode-3.sce

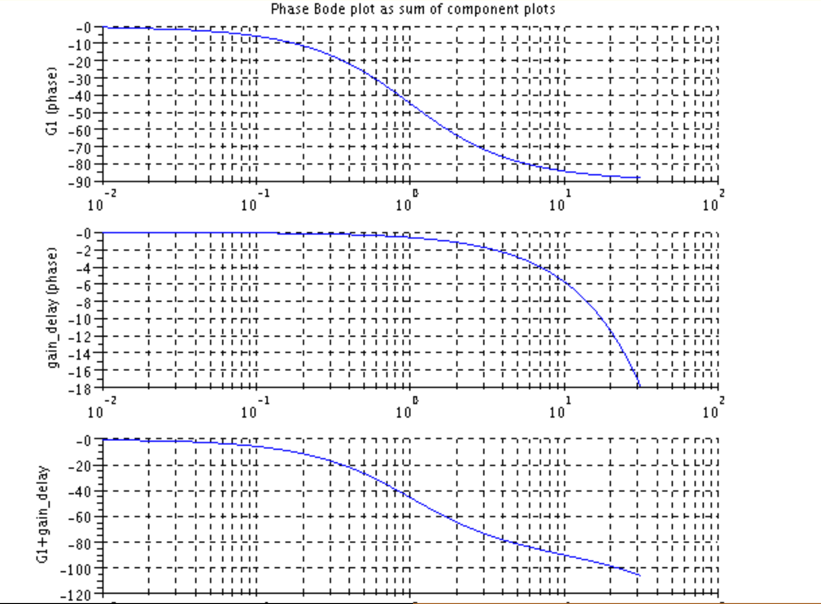
Bode plot of $G(s) = \frac{1}{s+1} e^{-0.01s}$

```
exec('bodesum-2.sci');
s = %s;
G1 = 1/(s+1);
gain = 1;
delay = 0.01;
w = 0.01:0.008*%pi:10*%pi;
bodesum_1(G1,delay,gain,w);
```

Magnitude Bode Plot



Phase Bode Plot



Guidelines for drawing Bode plots

- **Axes: log axis for abscissa and normal axis for ordinate**
- **For each component transfer function,**
  - **Draw the asymptotes**
  - **Locate the value at corner frequency**
  - **Connect approximately and complete the plots**
- **Add the component values**

Lecture 25
Stability Analysis through Bode Plots

Process Control
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Monday, 30 September 2013

Outline

1. **Self study of a second order underdamped system**
2. **Stability analysis**
3. **Gain margin and phase crossover frequency**
4. **Phase margin and gain crossover frequency**

1. Second order underdamped system

Homework: Bode plot of a second order underdamped system

Tutorial problem

Draw the bode plot of

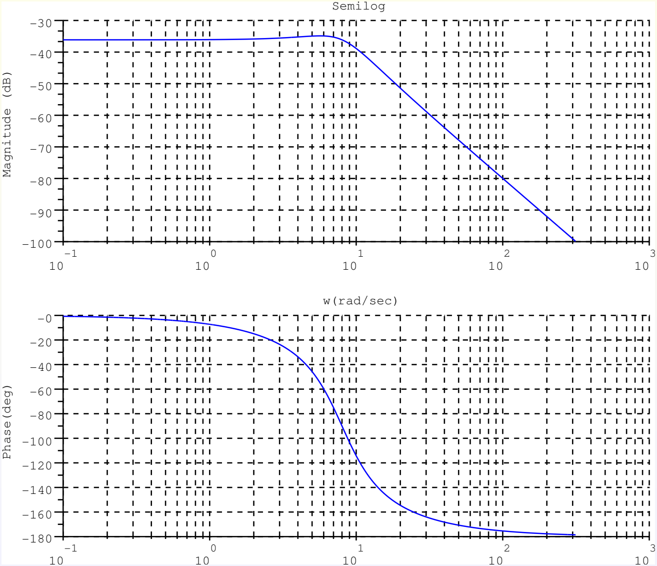
$$G(s) = \frac{1}{s^2 + 8s + 64}$$

- $\omega_n = 8, \zeta = 0.5$

Scilab code bode-6.sce

```
exec('bodegen-1.sci');

s = %s;
num = 1;
den = s^2+8*s+64;

w = 0.1:0.02:100*%pi;
LF = "semilog"

bodegen(num,den,w,LF);
```

Bode plot of an underdamped pole



2. Stability analysis using Bode plots

Instability Problem Statement

- G(s) is open loop transfer function
- Does not have poles and zeros on RHP
- Put in a closed loop with a proportional controller K_c
- As K_c increases, closed loop system becomes unstable
- We will first see the root locus plot conditions

Root locus stability conditions

- Root locus is the locus of roots of $1 + K_c G(s) = 0$, as K_c goes from 0 to ∞
- $1 + K_c G(s) = 0$ or $K_c G(s) = -1$
- Magnitude and phase relations:
- $|K_c G(s)| = 1$
  $\angle K_c G(s) = -180^\circ, \ +180^\circ, \ \pm 540^\circ, \ \text{etc.}$
- We will now see the conditions using Bode plot

Stability conditions for Bode plots

- To obtain Bode plots, substitute $s = j\omega$
- This corresponds to the imaginary axis of s plane
- Root locus conditions become,
  $|K_u G(j\omega)| = 1$,
  $\angle K_u G(j\omega) = -180^\circ, \pm 540$, etc.
- Because it is the boundary of instability, we have used K_u
- $K_c > K_u \Rightarrow$ closed loop system unstable
- Can analyse stability using Bode plot
- Can check by how much we can move
  - magnitude plot by adding gain
  - phase plot by adding delay

Restrict focus to class of systems

- **Restrict Bode plot analysis to a class of systems**
- **For $K_c < K_u$, system is stable**
- **For $K_c \geq K_u$, system is unstable**

Example

Find a proportional controller K_c that will make

$$G(s) = \frac{15}{(s+1)(s+2)(s+3)}$$

unstable, when put in a feedback loop.

Stability condition for example

- $1 + K_c G(s) = 0$
- $1 + \dfrac{15K_c}{(s+1)(s+2)(s+3)} = 0$
- $(s+1)(s+2)(s+3) + 15K_c = 0$
- $s^3 + 6s^2 + 11s + (15K_c + 6) = 0$
- Cuts imaginary axis at $K_c = 4$
- $K_u = 4$
- Stable for $K_c < 4$

Scilab code bode-10.sce

```
exec('bodedel.sci');

s = %s;
K = 1;
D = 0.1;
num = 15;
den = (s+1)*(s+2)*(s+3);
G = num/den;

w = 0.01:0.02:5;

bodedel(G,D,K,w);
```

Scilab code bodedel.sci I

```
// Bode plot with delay and gain

function bodedel(G1,delay,gain,w
   )

G1_freq = horner(G1,%i*w);
G1_mag = 20*log10(abs(G1_freq));
gain_freq = horner(gain,%i*w);
gain_mag = 20*log10(abs(
   gain_freq));

// xset('window',0); clf();
```

Scilab code bodedel.sci II

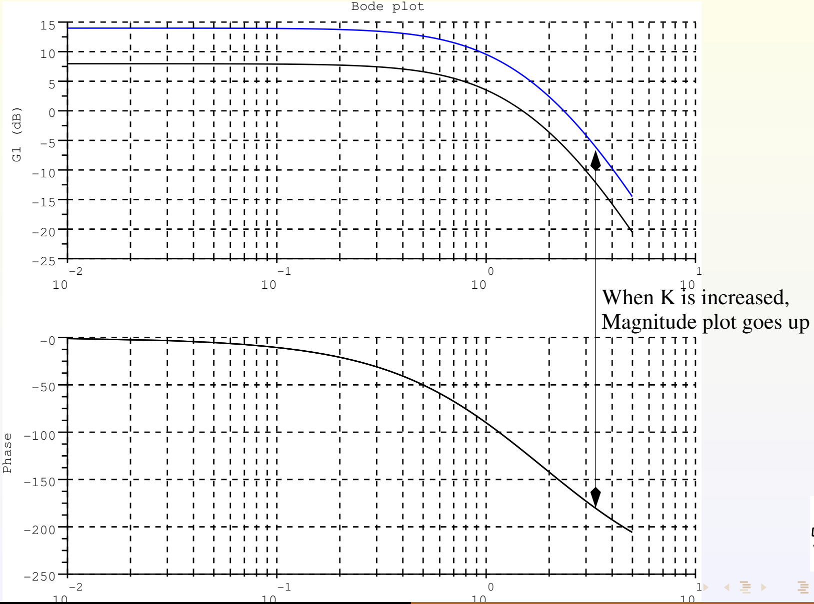
```

subplot(2,1,1)
xgrid();
xtitle('Bode plot','','G1 (dB)');
plot2d(w,G1_mag+gain_mag,logflag="ln",style = 1);

G1_ph = phasemag(G1_freq);
gain_ph = phasemag(gain_freq) - delay*w*180/%pi;

```

Scilab code bodedel.sci III

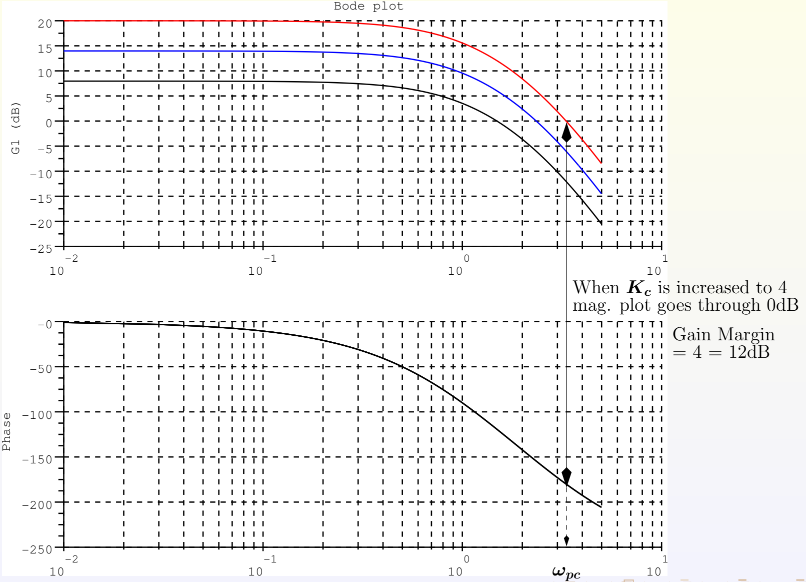
```
subplot(2,1,2)
xtitle('','w in rad','Phase');
plot2d(w,G1_ph+gain_ph,logflag="ln",style = 1);
xgrid();
endfunction;
```

3. Gain margin and phase crossover frequency

Increasing the gain



Increasing the gain further



Phase Crossover Frequency

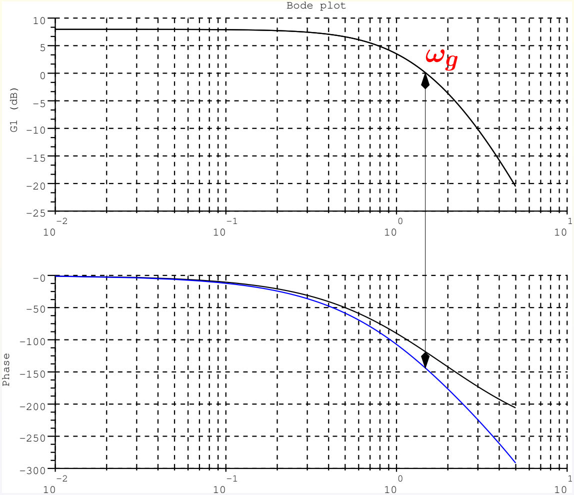
- The frequency ω at which $\angle G(j\omega) = -180^\circ$
- is called Phase Crossover Frequency
- It is denoted by ω_{pc}
- That is, $\angle G(j\omega_{pc}) = -180^\circ$
- Some people call it as simply crossover frequency, and denote it as ω_c

Gain Margin

- Locate ω_c, where $\angle G(j\omega_c) = -180^\circ$
- Find $|G(j\omega_c)|$ at that point
- Can increase gain of the system by K_c until $K_c|G(j\omega_c)| = 1$
- Can verify that we can increase K_c until 4
- Gain margin = 4 or 12 dB
- Draw the Bode plot and verify

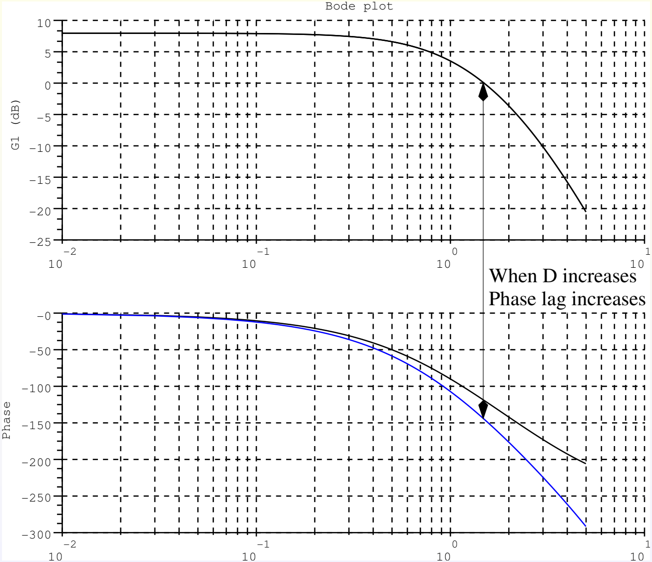
3. Phase margin and gain crossover frequency

Gain Crossover Frequency

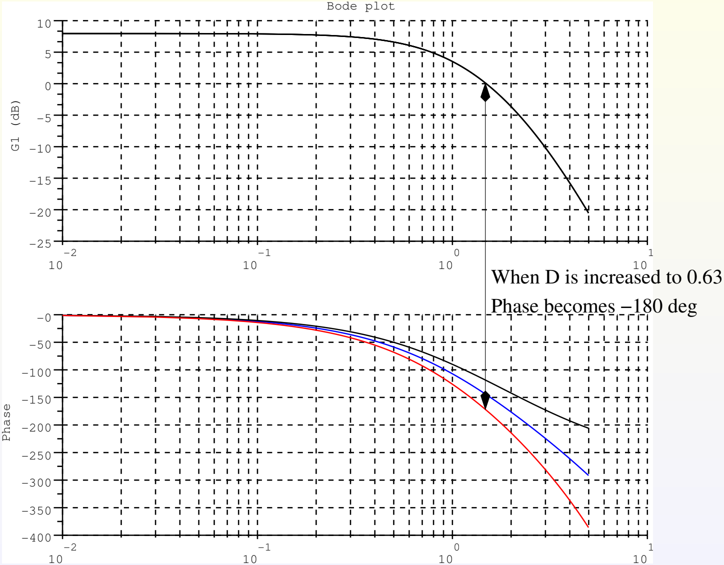


The frequency where $|G(j\omega)| = 0$ dB is called gain crossover frequency

Increasing the delay



Increasing the delay further



Bode plot by changing delay

- **Suppose G(s) changes to $G_1(s) = G(s)e^{-Ds}$**
- **What is D such that when $|G_1(j\omega)| = 1$, $\angle G_1(j\omega) = -180°$?**
- **Call this ω as ω_g, or gain crossover frequency**

Calculation of Delay

- $|G(j\omega_g)| = 1$
- $G(s) = \dfrac{15}{(s+1)(s+2)(s+3)}$
- $(\omega_g^2 + 1)(\omega_g^2 + 4)(\omega_g^2 + 9) = 225$
- $\omega_g \simeq 1.57$
- $\phi(j\omega_g) =$
  $-\tan^{-1}(\omega_g) - \tan^{-1}(\omega_g/2) - \tan^{-1}(\omega_g/3)$
- $= -123.2^\circ$
- If delay contributes -56.8° $(= 180 - 123.2^\circ)$, instability
- $D\omega_g = \dfrac{56.8}{180} \times \pi \Rightarrow D = 0.63$

Application to example

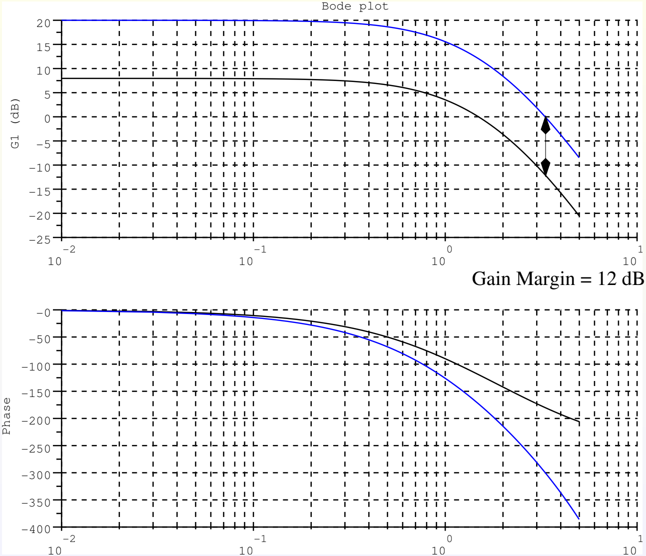
- **Can find $\omega_g = 1.57$, approximately**
- **Can increase D to $D = 0.63$**

Restrictions

This analysis is valid only for systems that have

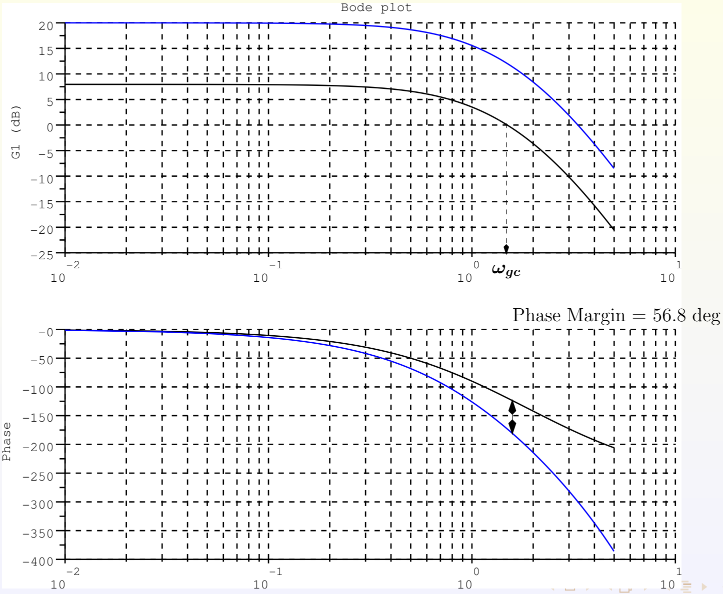
- **stable systems, with at most one pole on imaginary axis**
- **only one ω_c**
- **only one ω_g**

Gain margin



Bode plot
G1 (dB)
20
15
10
5
0
−5
−10
−15
−20
−25
10 −2
10 −1
10 0
10 1
Gain Margin = 12 dB
Phase
−0
−50
−100
−150
−200
−250
−300
−350
−400
10 −2
10 −1
10 0
10 1

Phase margin



Stabilising through derivative mode

What we learnt today

- **Stability conditions using Bode plot**
- **Stability margins**

Thank you