Lecture 26
Controller Design using Bode Plot

Process Control
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Outline

1. **Stability analysis using Bode plot**
2. **Example problem**
3. **Solution to example problem**

1. Summary of Stability using Bode Plots

Stability conditions for Bode plots

- Root locus conditions on $j\omega$ are
  $|K_u G(j\omega)| = 1$,
  $\angle K_u G(j\omega) = -180^\circ, \pm 540$, etc.
- Because it is the boundary of instability, we have used K_u
- $K_c > K_u \Rightarrow$ closed loop system unstable
- Can analyse stability using Bode plot
- Can check by how much we can move
  - magnitude plot by adding gain
  - phase plot by adding delay

Restrict focus to class of systems

- **Restrict Bode plot analysis to a class of systems**
- **For $K_c < K_u$, system is stable**
- **For $K_c \geq K_u$, system is unstable**

Example

Find a proportional controller K_c that will make

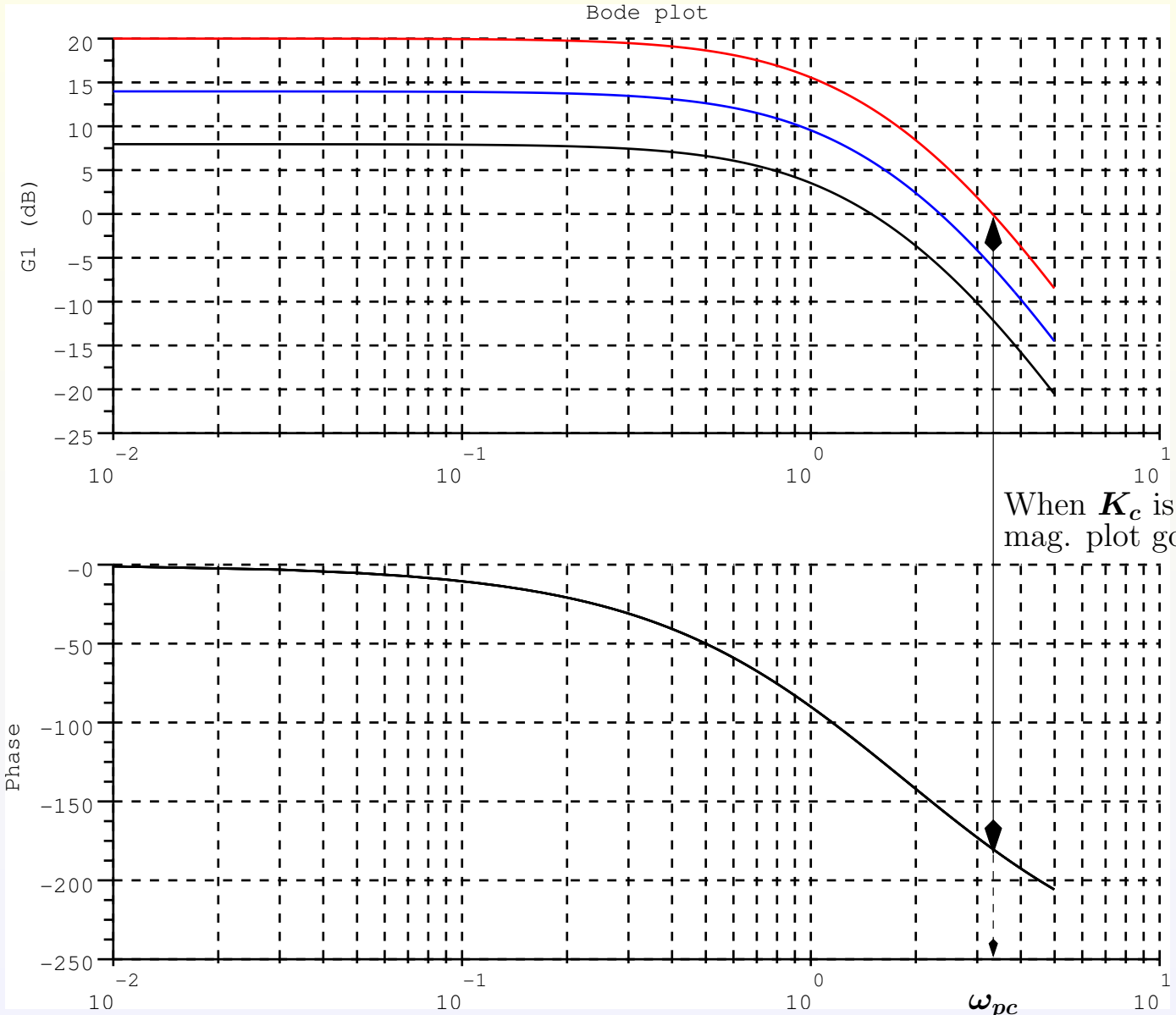
$$G(s) = \frac{15}{(s+1)(s+2)(s+3)}$$

unstable, when put in a feedback loop.

Stability condition for example

- $1 + K_c G(s) = 0$
- $1 + \dfrac{15K_c}{(s+1)(s+2)(s+3)} = 0$
- $(s+1)(s+2)(s+3) + 15K_c = 0$
- $s^3 + 6s^2 + 11s + (15K_c + 6) = 0$
- Cuts imaginary axis at $K_c = 4$
- $K_u = 4$
- **Stable for $K_c < 4$**

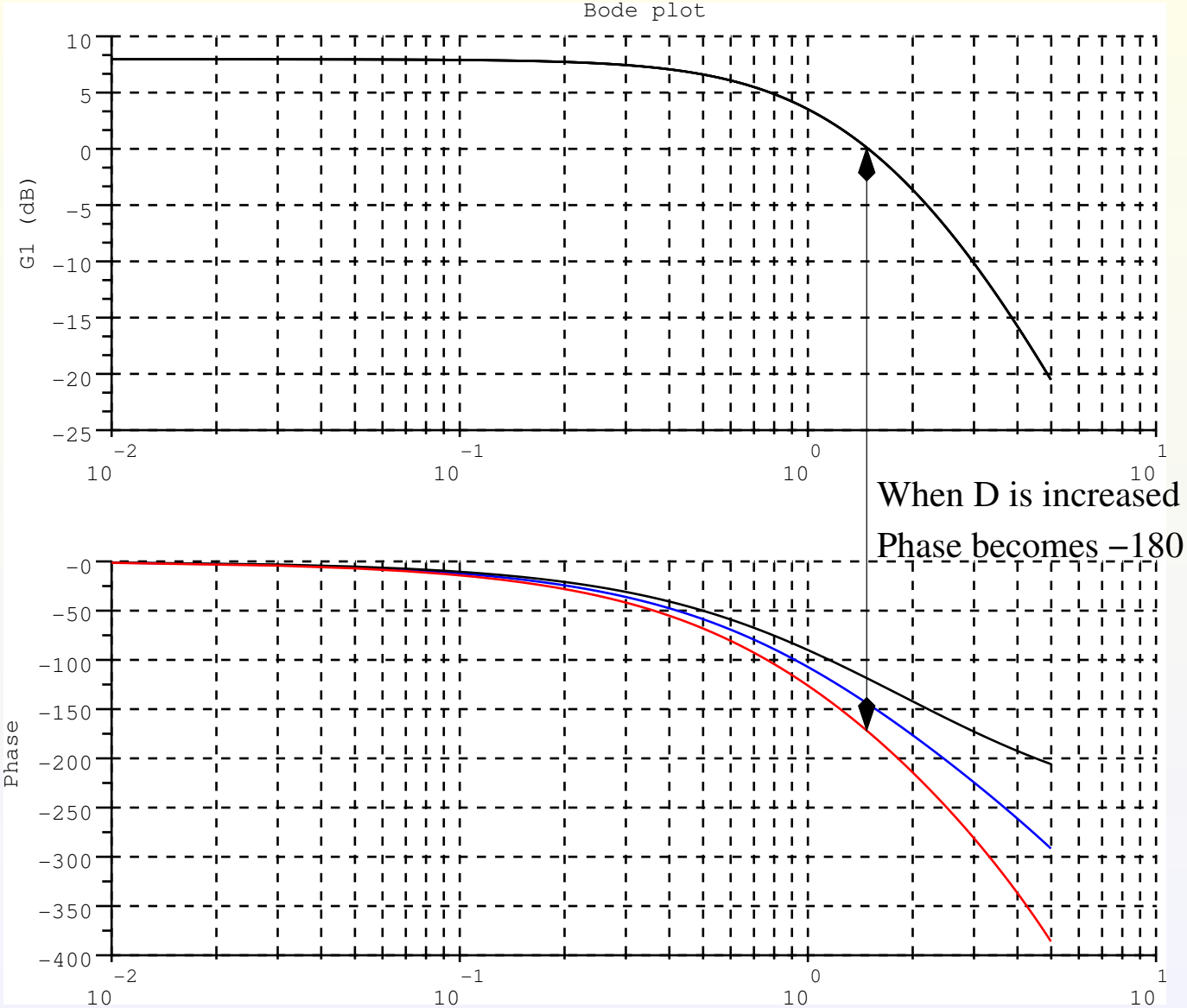
Example of Gain Margin



When K_c is increased to 4 mag. plot goes through 0dB

Gain Margin $= 4 = 12$dB

Example of Phase Margin



2. Example Problem

Example

Want to use $K_c = 10$ with this plant:

$$G(s) = \frac{15}{(s+1)(s+2)(s+3)}$$

MCQ: Choice of Kc

I want to use $K_c = 10$ because

1. **I can stabilise the system somehow**
2. **It is a round number**
3. **I want the steady state error to be small**
4. **This is done to illustrate a principle**

Answer: 3

Draw the root locus plot

$$G(s) = \frac{15}{(s+1)(s+2)(s+3)}$$

$$K_c = 10$$

Draw the Bode plot

$$G(s) = \frac{15}{(s+1)(s+2)(s+3)}$$

$$K_c = 10$$

3. Solution

Stabilised by increasing the phase at ω_g

MCQ: Adding Phase at ω_g

Phase can be added by

1. **Including a derivative mode, as noise effects are generally negligible**
2. **Better to include a lead, as a pure derivative mode cannot be implemented**
3. **Phase cannot be added**
4. **Adding a phase cannot stabilise the system**

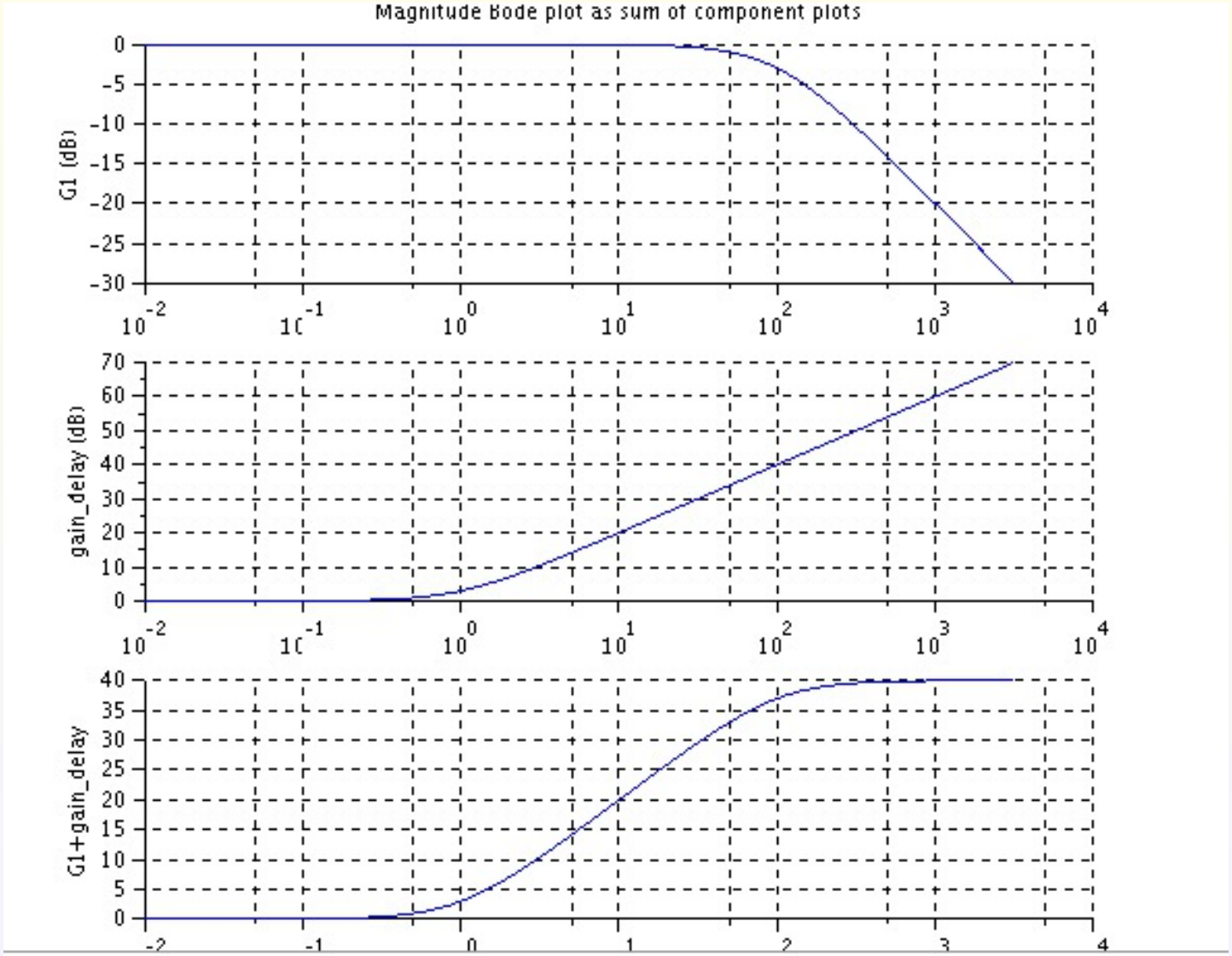
Answer: 2

Lead Transfer Function

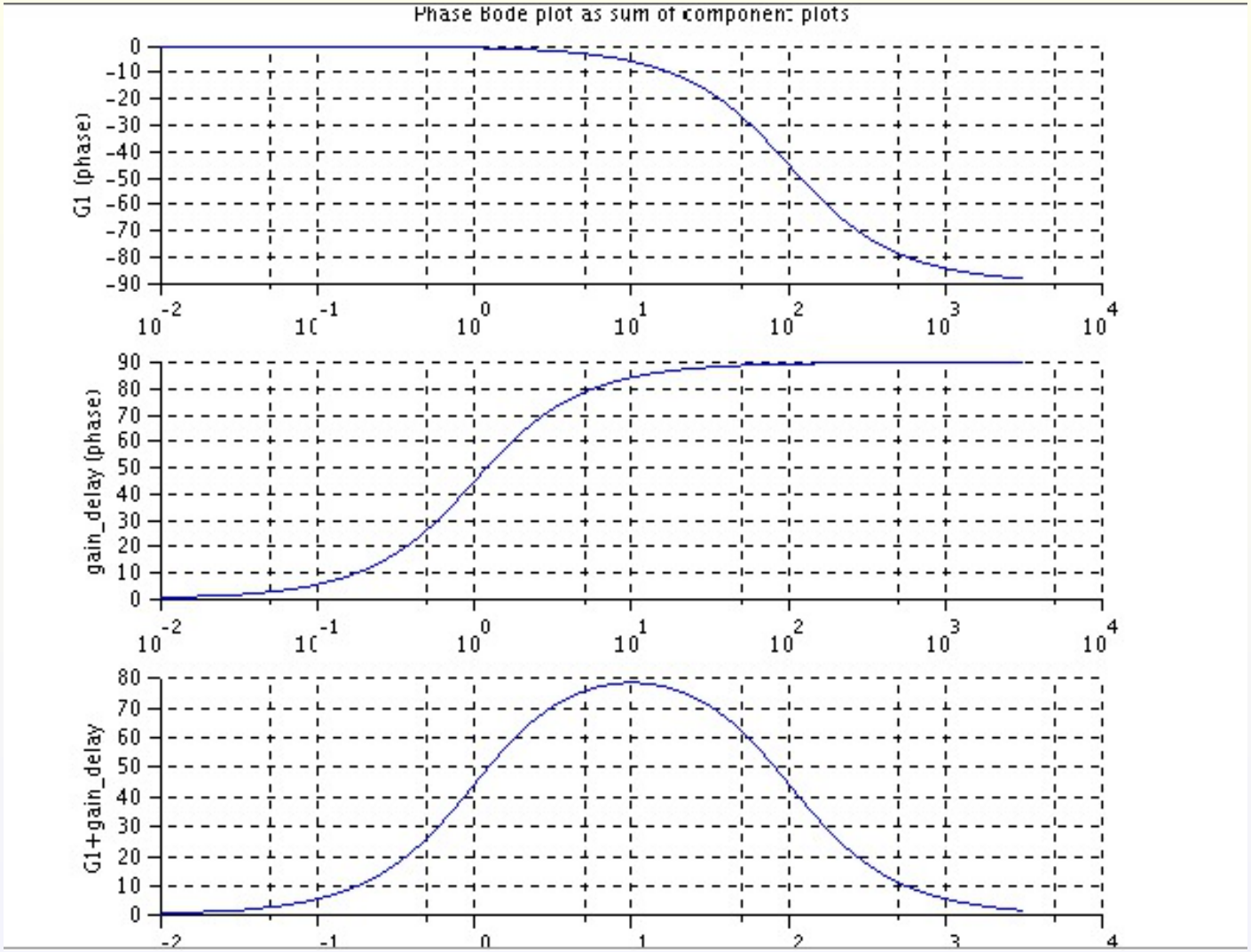
- **Consider the lead transfer function:**

$$G(s) = \frac{s+1}{0.01s+1}$$

Magnitude Bode Plot



Phase Bode Plot



Restrictions

This analysis is valid only for systems that have

- **stable systems, with at most one pole on imaginary axis**
- **only one ω_c**
- **only one ω_g**

What we learnt today

- **Solving a problem by trial and error**

Thank you