Lecture 32
Dynamic Matrix Control

Process Control

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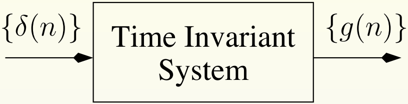
Outline

1. **Recap of discrete time control basics**
2. **Deriving the system matrix for DMC**
3. **Dynamic Matrix Control law**

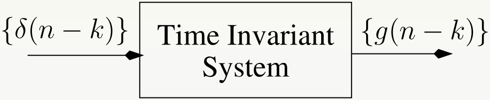
1. Recap of discrete time control basics

Impulse Response Models for LTI Systems

- **Recall impulse response:**



- **Shift the input. As time invariant, output will also be shifted:**



- $\{\mathbf{u(n)}\} = \sum_{\mathbf{k}=-\infty}^{\infty} \mathbf{u(k)}\{\delta(\mathbf{n}-\mathbf{k})\}$
- $\{\mathbf{y(n)}\} = \sum_{\mathbf{k}=-\infty}^{\infty} \mathbf{u(k)}\{\mathbf{g(n-k)}\}$
- **As before, use convolution operator, $*$:**
  $\{\mathbf{y(n)}\} = \{\mathbf{u(n)}\} * \{\mathbf{g(n)}\}$

Importance of Impulse Response Models

$$\{y(n)\} = \sum_{k=-\infty}^{\infty} u(k)\{g(n-k)\}$$

$$\triangleq \{u(n)\} * \{g(n)\}$$

- **Impulse response has all information about LTI system**
- **Given impulse response, can determine output due to any arbitrary input**

Step Response, Impulse Response

The unit step response of an LTI system at zero initial state $\{s(n)\}$ is the output when $\{u(n)\} = \{1(n)\}$:

$$\{s(n)\} = \sum_{k=-\infty}^{\infty} 1(k)\{g(n-k)\}$$

Apply the meaning of $1(k)$:

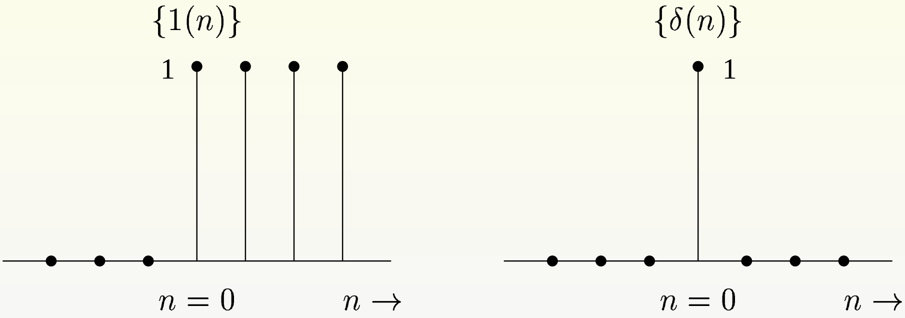
$$= \sum_{k=0}^{\infty}\{g(n-k)\}$$

- **This shows that the step response is the sum of impulse response.**

Relation between Step and Impulse Responses

- **We can also get impulse response from step response.**

Relation between Step and Impulse Signals



$$\{\delta(n)\} = \{1(n)\} - \{1(n-1)\}$$

Relation: Step and Impulse Responses

- We can also get impulse response from step response.
- $\{\delta(n)\} = \{1(n)\} - \{1(n-1)\}$
- Using linearity and time invariance properties,
- $\{g(n)\} = \{s(n)\} - \{s(n-1)\} = \triangle\{s(n)\}$
- Can show that $\{y(n)\} = [\{u(n)\} - \{u(n-1)\}] * \{s(n)\}$
- This can be written as $\{y(n)\} = \{\triangle u(n)\} * \{s(n)\}$
- Compare: $\{y(n)\} = \{u(n)\} * \{g(n)\}$

Superposition Principle

- **Output $y = y_u + y_x$**

Sum of output due to input only and output due to nonzero initial condition only.

2. Deriving the system matrix for DMC

DMC Modelling

- Let $s(0) = 0$, and a bias term b; it is for modelling errors, noise, etc. Don't know b
- $\hat{y}(k+1) = y_x(k+1) + s(1)\Delta u(k) + b(k+1)$
- $\hat{y}(k+2) = y_x(k+2) + s(2)\Delta u(k)$
  $+s(1)\Delta u(k+1) + b(k+2)$
- $\vdots$
- $\hat{y}(k+N_u+1) = y_x(k+N_u+1) + s(N_u+1)\Delta u(k)$
  $+ \cdots + s(1)\Delta u(k+N_u) + b(k+N_u+1)$

DMC Modelling, continued

- $\hat{y}(k+N_u+1) = y_x(k+N_u+1) + s(N_u+1)\Delta u(k) + \cdots + s(1)\Delta u(k+N_u) + b(k+N_u+1)$
- N_u is control horizon, i.e., we apply control effort up to $k+N_u$ and keep it constant afterwards
- i.e., $\Delta u(k+m) = 0$, for all $m > N_u$
- $\hat{y}(k+N_u+2) = y_x(k+N_u+2) + s(N_u+2)\Delta u(k) + \cdots + s(2)\Delta u(k+N_u) + b(k+N_u+2)$

DMC Modelling, continued

- $\hat{y}(k+N_u+2) = y_x(k+N_u+2) + s(N_u+2)\Delta u(k) + \cdots + s(2)\Delta u(k+N_u) + b(k+N_u+2)$
- $\vdots$
- $\hat{y}(k+N) = y_x(k+N) + s(N)\Delta u(k) + s(N-1)\Delta u(k+1) + \cdots + s(N-N_u)\Delta u(k+N_u) + b(k+N)$
- Stacking these up,
- $\underline{\hat{y}}(k+1) = \underline{y}_x(k+1) + \underline{s}\,\underline{u}(k) + \underline{b}(k+1)$

Definition of variables

$$\underline{\hat{y}}(k+1) = \begin{bmatrix} \hat{y}(k+1) \\ \hat{y}(k+2) \\ \vdots \\ \hat{y}(k+N) \end{bmatrix}, \quad \underline{y}_x(k+1) = \begin{bmatrix} y_x(k+1) \\ y_x(k+2) \\ \vdots \\ y_x(k+N) \end{bmatrix}$$

$$\underline{b}(k+1) = \begin{bmatrix} b(k+1) \\ b(k+2) \\ \vdots \\ b(k+N) \end{bmatrix}$$

Definition of variables: system matrix

$$\underline{s} = \begin{bmatrix} s(1) & & & \cdots & 0 \\ s(2) & s(1) & & \cdots & 0 \\ \vdots & & & & \\ s(N_u) & s(N_u-1) & s(N_u-2) & \cdots & 0 \\ s(N_u+1) & s(N_u) & s(N_u-1) & \cdots & s(1) \\ \vdots & & & & \\ s(N) & s(N-1) & s(N-2) & \cdots & s(N-N_u) \end{bmatrix}$$

The variable $\underline{s}$ has N rows and $N_u + 1$ columns, $N \geq N_u$

Definition of variables: control effort

$$\underline{u}(k) = \begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \vdots \\ \Delta u(k+N_u) \end{bmatrix}$$

$\underline{u}$ has $N_u + 1$ rows

Assignment

Develop the system matrix for $N = 5$, $N_u = 3$

3. Dynamic matrix control law

Solution to DMC model

- $\underline{r}(k+1) = \begin{bmatrix} r(k+1) & r(k+2) & \cdots & r(k+N) \end{bmatrix}^T$
- Letting $\underline{\hat{y}} = \underline{r}$, we obtain,
- $\underline{y}_x(k+1) + \underline{s}\,\underline{u}(k) + \underline{b}(k+1) = \underline{r}(k+1)$
- By rearranging the terms, we require
- $\underline{s}\,\underline{u}(k) - \left[\underline{r}(k+1) - \underline{y}_x(k+1) - \underline{b}(k+1)\right] = 0$
- Defining the terms within square brackets as $\underline{e}(k+1)$,
- $\underline{s}\,\underline{u}(k) - \underline{e}(k+1) = 0$
- Least squares solution:
- $\underline{u}(k) = (\underline{s}^T\underline{s})^{-1}\underline{s}^T\underline{e}(k+1)$

DMC: control law

- $\underline{u}(k) = (\underline{s}^T\underline{s})^{-1}\underline{s}^T\underline{e}(k+1)$
- In order that excessive control action is not applied, the following method of control action calculation is done:
- $\underline{u}(k) = \left[\underline{s}^T\underline{s} + \rho^2 I\right]^{-1}\underline{s}^T\underline{e}(k+1)$
- We determine the current bias and assume that it is constant for the rest of the control moves
- In other words, we let $b(k+1) = y(k) - \hat{y}(k),\ i = 1, 2, \ldots, N$, where y is the measured value of output.

Receding Horizon Control

- **Implement only the first control law**
- **Repeat the calculations for every control move**
- **Calculate the bias term for every interval, using the latest measured y value**

Reference

Reference for Digital Control:
K. M. Moudgalya, Digital Control, Wiley, Chichester, 2007
Also, New Delhi, 2009

What we learnt today

- **Recap of discrete time model**
- **DMC model**

Thank you