Lecture 8
Transfer Function Definition
Block Diagram Manipulation

Process Control
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Outline

1. **Formal definition of transfer functions**
2. **Block diagram manipulation**
3. **Time delay**

1. Formal definition of transfer functions

One Definition of Impulse Function, δ

- **It is a function with a nonzero value at only one point, with area unity**
- **Rectangle of infinite height with zero width**
- **Triangle of infinite height with zero base length**

Second Definition of Impulse Function

- **Let the impulse function be $\delta(t)$**
- **The integral**

$$\int_a^b f(t)\delta(t)dt = \begin{cases} f(0), & a < 0 < b \\ 0, & 0 \notin (a, b) \end{cases}$$

- **The Laplace Transform of $\delta(t)$ is**

$$\int_0^\infty e^{-st}\delta(t)dt = 1$$

Transfer function

Recall the procedure we used to derive transfer function:

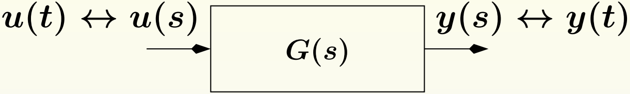
- **Considered only time invariant systems**
- **Linearised them**
- **Made initial conditions to be zero**

Impulse response

- **We studied step response, ramp response, etc.**
- **The impulse response of a system is**
  1. **step response to a system which behaves like an impulse**
  2. **inverse Laplace transform of its transfer function**
  3. **don't know what it is**

Definition of transfer functions

- **Recall the input-output property**



- **What will be the output, if the input is impulse?**
- **y(s) = G(s)u(s) = G(s), in case of impulse input**
- **i.e. y(s) = G(s), in case of impulse input**
- **So, g(t) (where, g(t) ↔ G(s)) is known as the impulse response**
- **Transfer function is the Laplace Transform of impulse response**

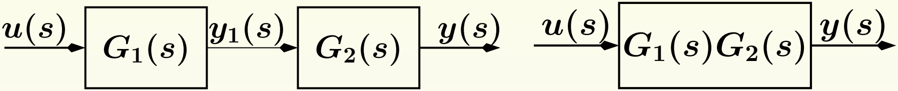
Impulse response

- **We studied step response, ramp response, etc.**
- **The impulse response of a system is**
  1. **step response to a system which behaves like an impulse**
  2. **inverse Laplace transform of its transfer function**
  3. **don't know what it is**

Answer: 2

2. Block diagram manipulation

Transfer functions in series

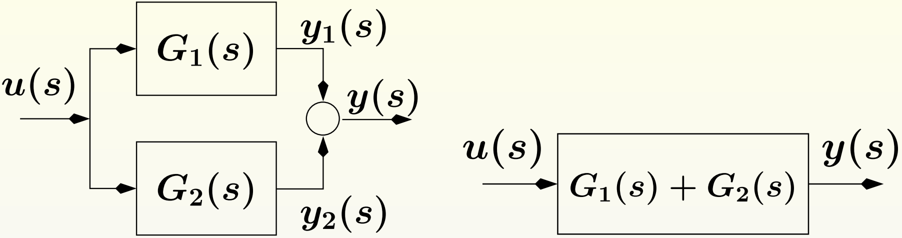


- $y_1(s) = G_1(s)u(s)$
- Under what conditions? Initial condition is zero
- Or when deviational variables are used
- Also when the model is linear
- $y(s) = G_2(s)y_1(s) = G_2(s)G_1(s)u(s)$
- Overall transfer function $= G_2(s)G_1(s)$
- $= G_1(s)G_2(s)$, in case of scalars

A good counter example

- **When nonlinear or time varying, systems do not commute**
- **J. C. Proakis and D. G. Manolakis, Digital Signal Processing Principles, Algorithms, and Applications. Prentice Hall, Inc., Upper Saddle River, NJ and also New Delhi**
- **Repeated in K. M. Moudgalya, Digital Control. John Wiley & Sons, Chichester and also New Delhi**

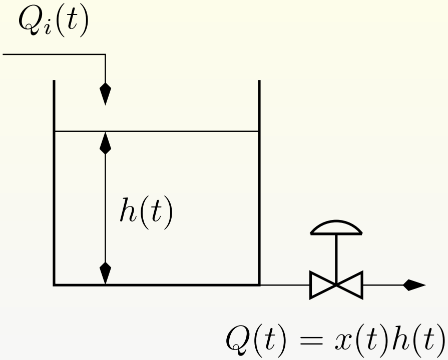
Transfer functions in parallel



- $y_1(s) = G_1(s)u(s)$
- Once again, under zero initial conditions
- Or when deviational variables are used
- $y(s) = y_1(s) + y_2(s) = G_1(s)u(s) + G_2(s)u(s)$
- $= (G_1(s) + G_2(s))u(s)$
- Overall transfer function $= G_1(s) + G_2(s)$

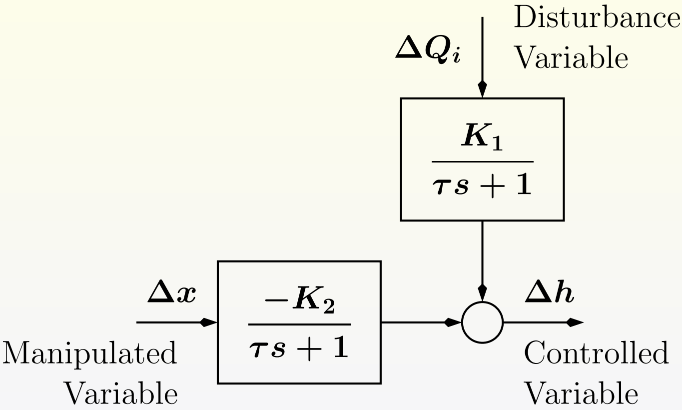
Closed loop transfer function

Recall: Model of Flow Control System



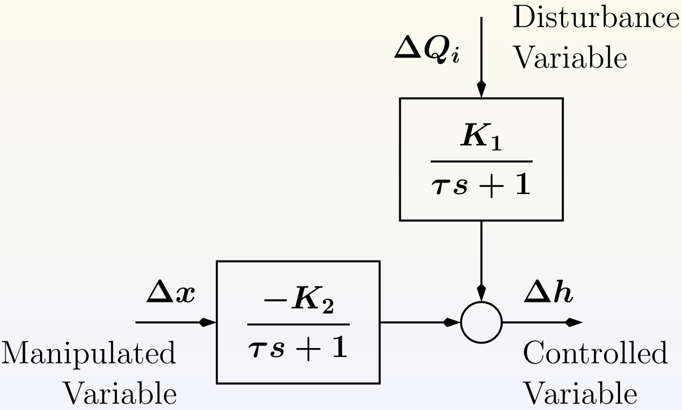
$$\Delta h(s) = \frac{K_1}{\tau s + 1}\Delta Q_i(s) - \frac{K_2}{\tau s + 1}\Delta x(s)$$

Recall the block diagram representation

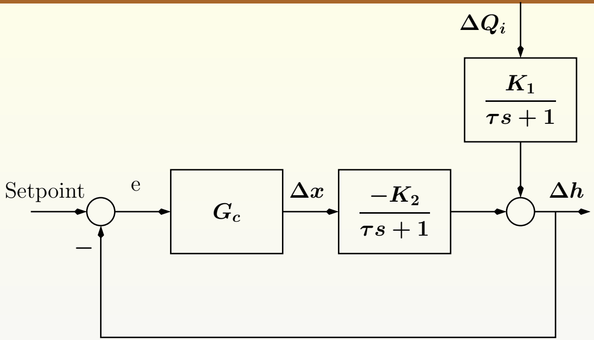


- **+ sign is not explicitly shown**
- **− sign has to be shown at the summing junction**

How do we use this model in feedback control design?

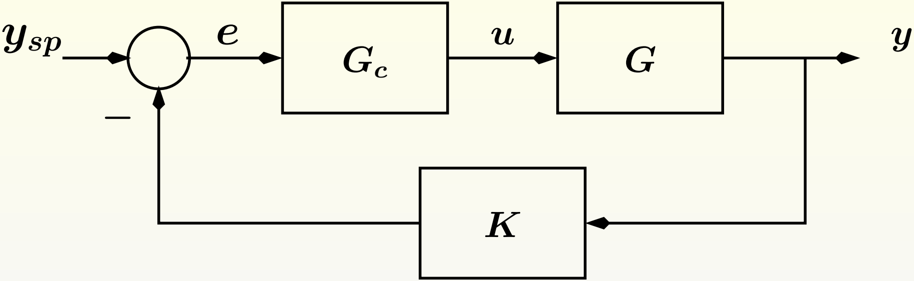


Feedback control system block diagram



- G_c is controller, to design in this course
- Setpoint is the desired value of height
- e is the error
- The output is subtracted from the setpoint
- What is the relation between setpoint and Δh?

A simplified closed loop transfer function

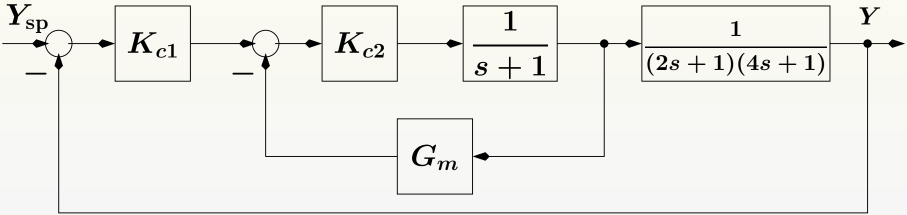


- What is the relationship between y and y_{sp}?
- K is the measurement transfer function
- G_c is the transfer function of the controller
- The closed loop relation is

$$y(s) = \frac{GG_c}{1 + KGG_c}$$

Problem from Final Exam, 2009

This problem is concerned with a scheme, known as cascade control, shown below:



Determine the closed loop transfer function

3. Time delay

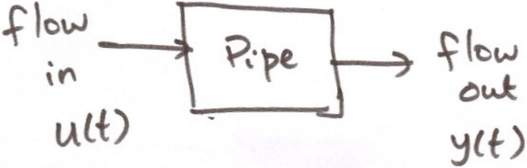
Time Shift Laplace Transform

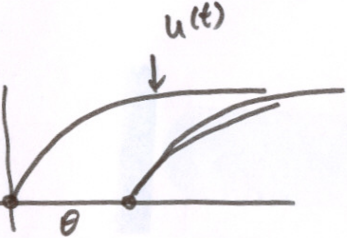
- **Let the Laplace Transform of $f(t)$ be $F(s)$**
- **The Laplace Transform of $f(t-L)$ is**
  1. $F(s-L)$
  2. $e^{L}F(s)$
  3. $e^{-L}F(s)$
  4. $e^{-sL}F(s)$
- **Answer:** $e^{-sL}F(s)$
- **So, the Laplace Transform of** $u(t) = [1(t) - 1(t-t_1)] \times M$ **is**
- $U(s) = \left[\frac{1}{s} - \frac{1}{s}e^{-t_1 s}\right] M$

Time Delay Modelling

From a previous lecture

Time delay.





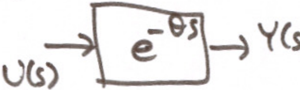
$$\boxed{y(t) = u(t - \theta)} \quad ①$$

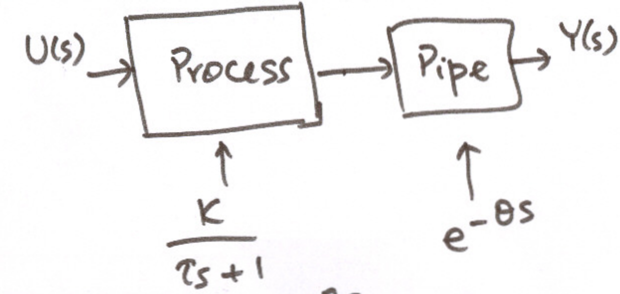
θ : residence time

L.T of Eq. 1

$u(t) \rightarrow U(s),\ y(t) \rightarrow Y(s$

$$Y(s) = U(s)\, e^{-\theta s}$$



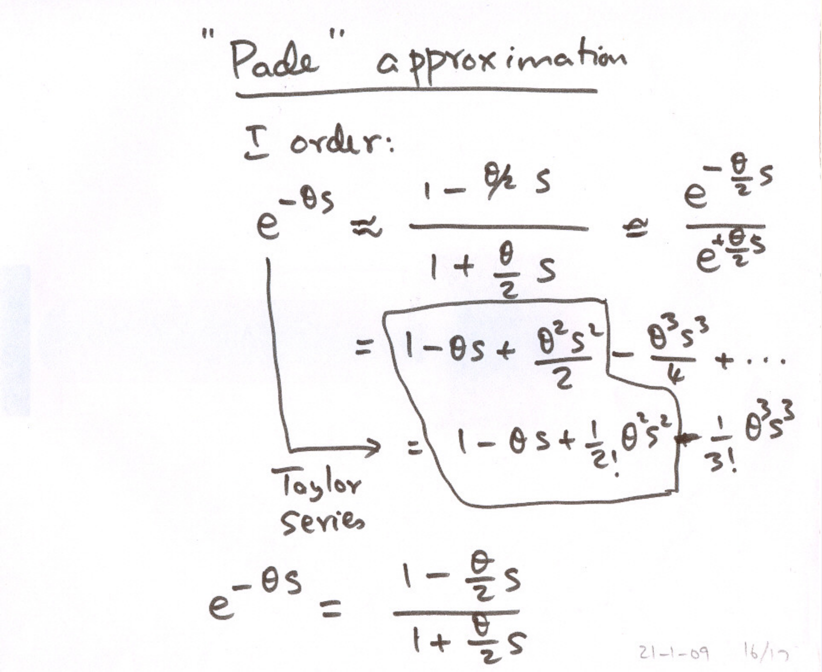


$$Y(s) = \frac{K e^{-\theta s}}{\tau s+1} U(s)$$

not a polynomial.

We carry out approximations

" Pade " approximation.



"Pade" approximation

I order:

$$e^{-\theta s} \approx \frac{1-\theta/2\, s}{1+\frac{\theta}{2}s} \hat{=} \frac{e^{-\frac{\theta}{2}s}}{e^{+\frac{\theta}{2}s}}$$

$$= 1-\theta s+\frac{\theta^2 s^2}{2} - \frac{\theta^3 s^3}{4} + \dots$$

Taylor series

$$= 1-\theta s + \frac{1}{2!}\theta^2 s^2 - \frac{1}{3!}\theta^3 s^3$$

$$e^{-\theta s} = \frac{1-\frac{\theta}{2}s}{1+\frac{\theta}{2}s}$$

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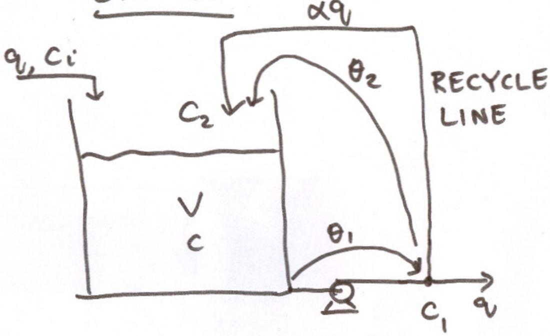
Ex. 6.3



$$\Delta C_1(s) = \frac{2000\, k e^{-\theta_1 s}}{s\left[\tau s + 1 + \alpha k\left(1 - e^{-\theta_3 s}\right)\right]}$$

$$c_1(s) = c(s)\, e^{-\theta_1 s}$$

$$c_2(s) = c(s)\, e^{-(\theta_1 + \theta_2)s}$$

Pade approximation used in the denominator.

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One Way to Determine Time Delay

- **Change the input by a step**
- **Find out when the output starts changing**
- **Determine the time delay**
- **Check whether the SBHS has any time delay!**

What we learnt today

- **Formal definition of transfer function**
- **Introduction to block diagram manipulation**
- **Time delay processes**

Thank you