Control Systems: Higher-order Systems

Learning Objectives

- General closed-loop response function
- Pole-zero map
  - -overdamped closed-loop poles
  - -critically damped closed-loop poles
  - -undamped closed-loop poles
  - -negatively underdamped closed-loop poles
  - -negatively overdamped closed-loop poles
- Transient response using residues
- Example

Control Systems: Higher-order Systems

General closed-loop response function

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

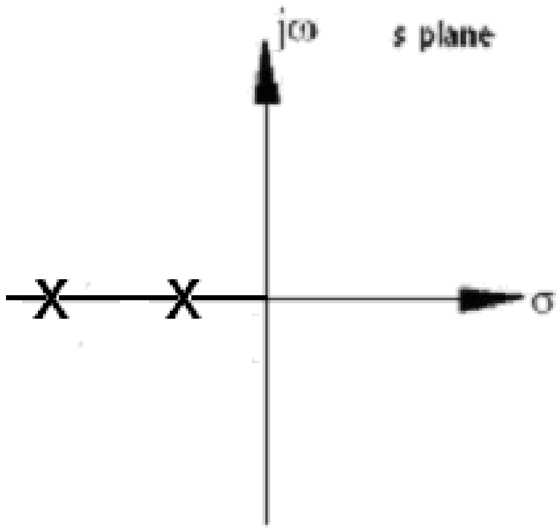
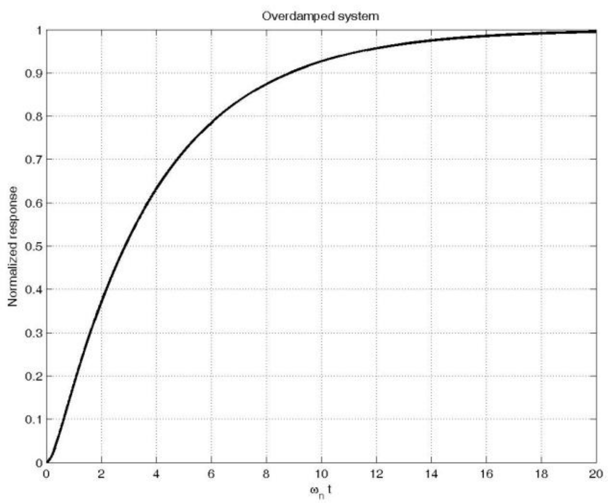
$$G(s)H(s) = \frac{K(s - z_1)(s - z_2)\ldots(s - z_m)}{(s - p_1)(s - p_2)\ldots(s - p_n)}$$

Open-loop transfer function

$$\frac{C(s)}{R(s)} = \frac{G(s)(s - p_1)(s - p_2)\ldots(s - p_n)}{(s - p_1)(s - p_2)\ldots(s - p_n) + K(s - z_1)(s - z_2)\ldots(s - z_m)}$$

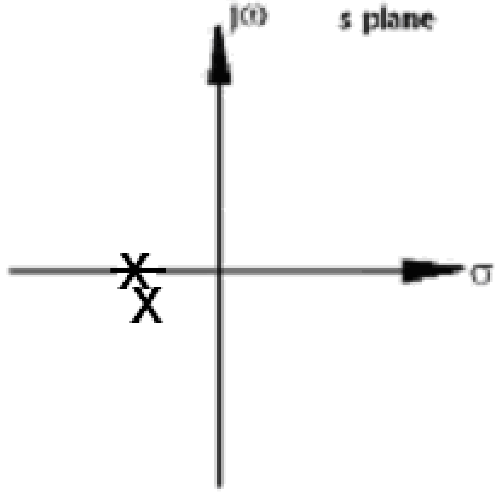
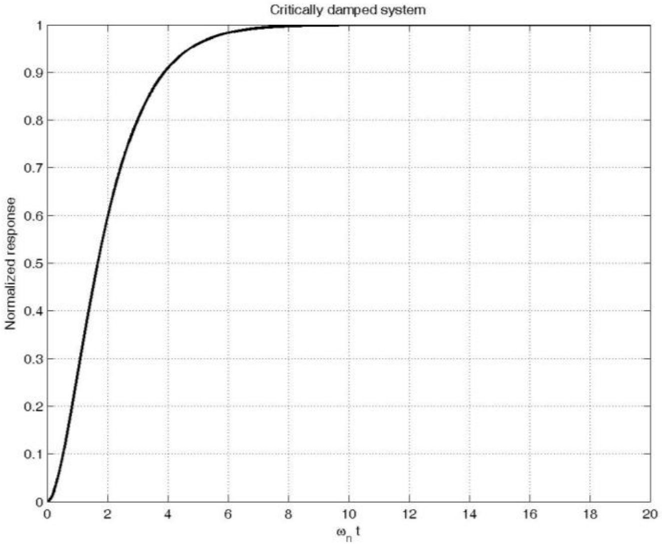
Control Systems: Higher-order Systems

Pole-zero map | Overdamped closed-loop poles

Pole-zero map	Normalized response
 (a)	 (b)

Control Systems: Higher-order Systems

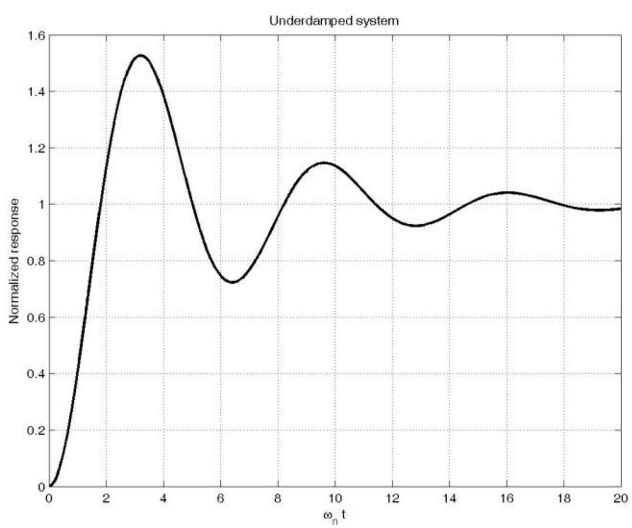
Pole-zero map | **Critically damped closed-loop poles**

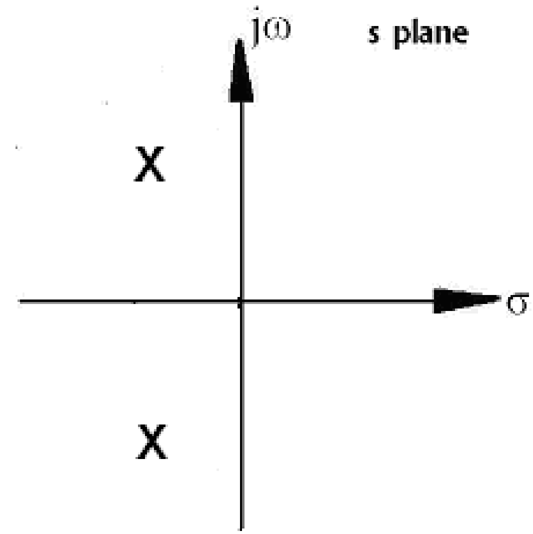
Pole-zero map	Normalized response
 (a)	 (b)

Control Systems: Higher-order Systems

Pole-zero map

Underdamped closed-loop poles

<table>
<tr><th>Normalized response</th><th>Pole-zero map</th></tr>
<tr><td>

(b)</td><td>

(a)</td></tr>
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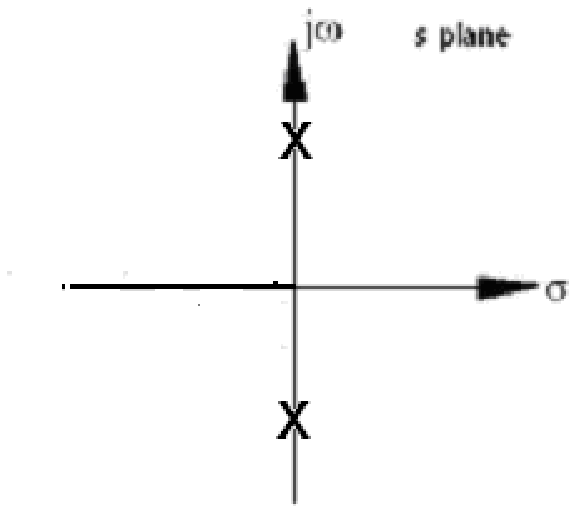
Control Systems: Higher-order Systems

Pole-zero map | **Undamped closed-loop poles**

<table>
<tr><th>Pole-zero map</th><th>Normalized response</th></tr>
<tr><td>


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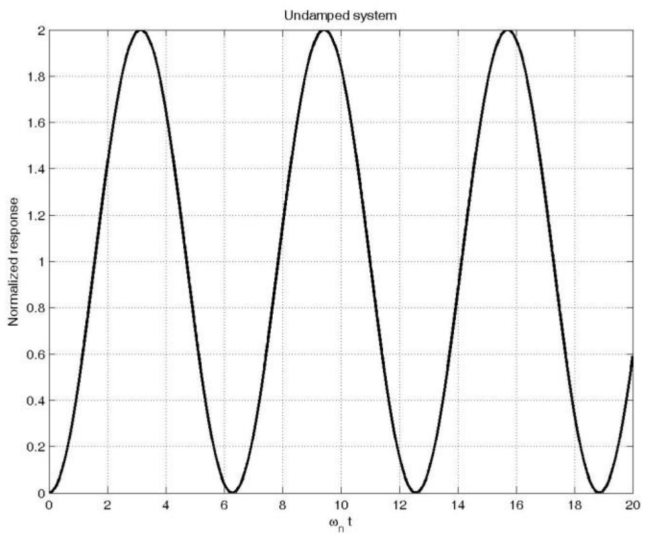
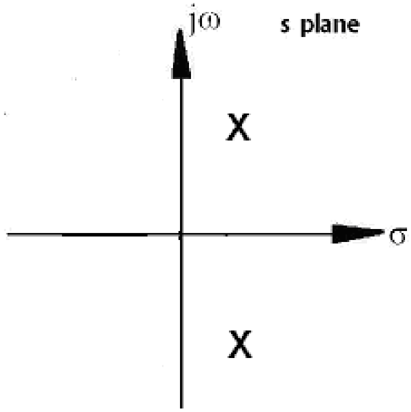
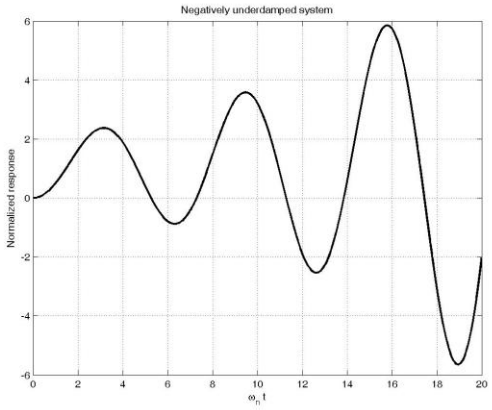


(b)
</td></tr>
</table>

Control Systems: Higher-order Systems

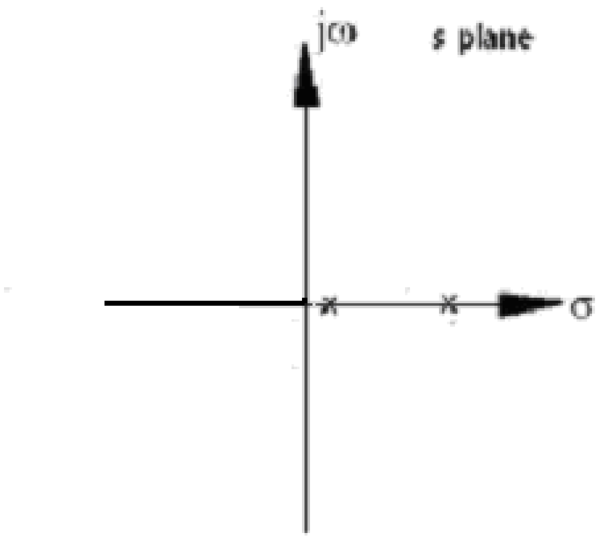
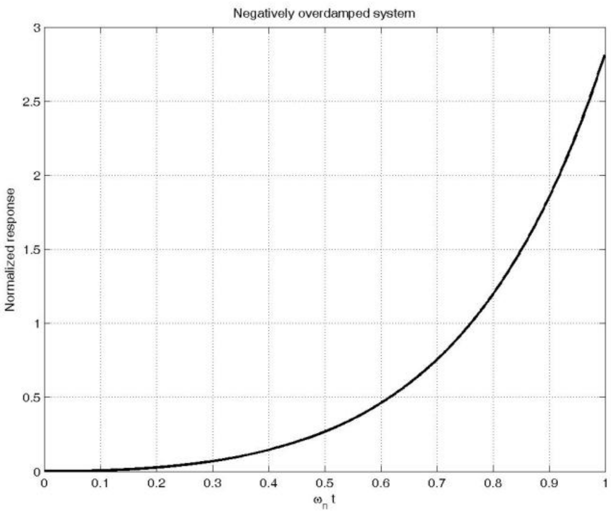
Pole-zero map **Negatively underdamped closed-loop poles**

Pole-zero map	Normalized response
 (a)	 (b)

Control Systems: Higher-order Systems

Pole-zero map

Negatively overdamped closed-loop poles

Pole-zero map	
	

Control Systems: Higher-order Systems

Transient response using residues

$$C(s) = \frac{R(s)G(s)(s-p_1)(s-p_2)\ldots(s-p_n)}{(s-p_1)(s-p_2)\ldots(s-p_n)+K(s-z_1)(s-z_2)\ldots(s-z_m)}$$

$$C(s) = \frac{b}{s} + \sum_{i=1}^{n} \frac{a_i}{s+p_i^c}$$

Step response

Control Systems: Higher-order Systems

Transient response using residues

$$a_i = \frac{G(s)(s-p_1)(s-p_2)\ldots(s-p_n)}{s\left[(s-p_1)(s-p_2)\ldots(s-p_n)+K(s-z_1)(s-z_2)\ldots(s-z_m)\right]}\times(s-p_i^c)\Bigg|_{s=p_i^c}$$

$$b = \frac{G(s)(s-p_1)(s-p_2)\ldots(s-p_n)}{s\left[(s-p_1)(s-p_2)\ldots(s-p_n)+K(s-z_1)(s-z_2)\ldots(s-z_m)\right]}\times s\Bigg|_{s=0}$$

$$c(t) = b + \sum_{i=1}^{n} a_i e^{p_i^c t}$$

Control Systems: Higher-order Systems

Example

$$G(s) = \frac{K_1}{(s+5)} \qquad H(s) = \frac{K_2}{(s+1)}$$

K_1=1 K_2 =5

$$G(s)H(s) = \frac{K_1 K_2}{(s+5)(s+1)} = \frac{K}{(s+5)(s+1)}$$

$$\frac{C(s)}{R(s)} = \frac{K_1(s+1)}{(s+5)(s+1)+K_1 K_2}$$

$$\frac{C(s)}{R(s)} = \frac{(s+1)}{(s+5)(s+1)+5} = \frac{s+1}{s^2+6s+10}$$

$$\boxed{p_1^c = -3 + j}$$

$$\boxed{p_2^c = -3 - j}$$

Example

Unit step input

$$C(s) = \frac{s+1}{s\left(s^2+6s+10\right)}$$

$$b = \frac{(s+1)}{s\left(s^2+6s+10\right)} \times s \Bigg|_{s=0} = 0.1$$

$$a_1 = \frac{(s+1)}{s\left(s^2+6s+10\right)} \times (s+3-j) \Bigg|_{s=-3+j} = \frac{(-2+j)}{(-3+j)(2j)} = -0.05-0.35j$$

$$a_2 = \frac{(s+1)}{s\left(s^2+6s+10\right)} \times (s+3+j) \Bigg|_{s=-3-j} = \frac{(-2-j)}{(-3-j)(-2j)} = -0.05+0.35j$$

Example

$$C(s) = \frac{0.1}{s} + \frac{(-0.05 - 0.35j)}{s+3-j} + \frac{(-0.05 + 0.35j)}{s+3+j}$$

$$c(t) = 0.1 + e^{-3t}(-0.1\cos t + 0.7\sin t)$$

Control Systems: Higher-order Systems

Example

$$G_1(s) = \frac{\dfrac{K_1}{(s+5)}}{1+\left(\dfrac{K_1}{s+5}\right)\left(\dfrac{K_2}{s+1}\right)-\dfrac{K_1}{(s+5)}} = \frac{1}{s^2+5s+9}$$

$$K_p = \lim_{s\to 0} G_1(s) = \frac{1}{9} \qquad s_e(\infty) = \frac{1}{1+K_p} = \frac{1}{1+\dfrac{1}{9}} = 0.9$$

$$c(t) = 0.1 + e^{-3t}(-0.1\cos t + 0.7\sin t)$$

ME 779 Control Systems

Topic #11

Synthetic Division

Reference textbook:

Control Systems, Dhanesh N. Manik, Cengage Publishing, 2012

Control Systems: Synthetic Division

Learning Objectives

- Closed-loop poles and roots of a polynomial
- Newton-Raphson’s method
- Limitations of Newton-Raphson’s method
- Dividing a polynomial
- Synthetic division

Control Systems: Synthetic Division

Closed-loop poles and roots of a polynomial

- The characteristic equation for closed-loop response is a polynomial
- The polynomial order is dependent on the number of closed-loop poles
- Solving the characteristic equation gives all the closed-loop poles
- Closed-loop poles determine the dominant behaviour of closed-loop response

Control Systems: Synthetic Division

Newton-Raphson's method

How to solve for the roots of a polynomial?

Using Newton-Raphson's method

$$f(s) = a_n s^n + a_{n-1} s^{n-1} + \cdots a_1 s + a_0$$

Using an initial guess of s_n

$$s_{n+1} = s_n - \frac{f(s_n)}{f'(s_{n)}}$$

Converges in 4 to 5 iterations

Control Systems: Synthetic Division

Limitations of Newton-Raphson's method

- For a higher-degree polynomial, guessing different starting points for every root is difficult; might converge to the same root from different starting points.
- Applying Newton-Raphson's method to compute all the roots, without reducing the polynomial order is computationally inefficient
- Therefore, the polynomial order should be reduced using a known root using synthetic division, before proceeding to computing the next root

Control Systems: Synthetic Division

Dividing a polynomial

$$
\begin{array}{r|l}
 & s^2 \\
\hline
s-2 & s^3 + 6s^2 + 4s + 2 \\
 & \underline{s^3 - 2s^2} \\
 & \qquad 8s^2 + 4s + 2
\end{array}
$$

Control Systems: Synthetic Division

Dividing a polynomial

$$
\begin{array}{r|l}
 & s^2 + 8s \\
\hline
s - 2 & s^3 + 6s^2 + 4s + 2 \\
 & \underline{s^3 - 2s^2} \\
 & \quad 8s^2 + 4s + 2 \\
 & \quad \underline{8s^2 - 16s} \\
 & \qquad 20s + 2
\end{array}
$$

Control Systems: Synthetic Division

Dividing a polynomial

$$
\begin{array}{r|l}
 & s^2 + 8s + 20 \quad \boxed{\textbf{Quotient}} \\
\hline
s - 2 & s^3 + 6s^2 + 4s + 2 \\
 & s^3 - 2s^2 \\
\hline
 & \quad 8s^2 + 4s + 2 \\
 & \quad 8s^2 - 16s \\
\hline
 & \qquad 20s + 2 \\
 & \qquad 20s - 40 \\
\hline
 & \qquad\quad 42 \quad \boxed{\textbf{Remainder}}
\end{array}
$$

Control Systems: Synthetic Division

Synthetic Division

Let us now obtain the quotient and remainder of the polynomial

$$s^3 + 6s^2 + 4s + 2$$ **when divided by** $$s - 2$$

1	6	4	2
	2		
1 $\xrightarrow{\times 2}$	8		

Coefficients of the polynomial

Control Systems: Synthetic Division

Synthetic Division

Let us now obtain the quotient and remainder of the polynomial

$s^3 + 6s^2 + 4s + 2$ when divided by $s - 2$

1	6	4	2
	2	16	
1 x2	8 x2	20	

Coefficients of the polynomial

Control Systems: Synthetic Division

Synthetic Division

Let us now obtain the quotient and remainder of the polynomial

$s^3 + 6s^2 + 4s + 2$ **when divided by** $s - 2$

1	6	4	2
	2	16	40
1	8	20	42

x2 x2 x2

Coefficients of the polynomial

Synthetic Division

Let us now obtain the quotient and remainder of the polynomial

$s^3 + 6s^2 + 4s + 2$ when divided by $s - 2$

1	6	4	2
	2	16	40
1	8	20	42

x2 x2 x2

Coefficients of the polynomial

Remainder

$$\underbrace{s^2 + 8s + 20}$$

Quotient

Control Systems: Synthetic Division

Synthetic Division

DivisorxQuotient+Remainder=Dividend

$$(s-2)(s^2+8s+20)+42=s^3+6s^2+4s+2$$

$$(s-r_1)(s^2+as+b)=s^3+6s^2+4s+2$$

Where r_1 is one of the roots of the polynomial, the remainder is zero and the quotient can be solved for the remaining roots; the values of the quotient a and b can be obtained from synthetic division

Control Systems: Synthetic Division

Example

Now let us obtain all roots of the polynomial by using Newton-Raphson's method and synthetic division

$$f(s) = s^3 + 6s^2 + 4s + 2$$

$$f'(s) = 3s^2 + 12s + 4$$

$$s_{n+1} = s_n - \frac{f(s_n)}{f'(s_n)}$$

Newton-Raphson's method

Control Systems: Synthetic Division

Example

Iteration No.	s_n	$f(s_n)$	$f'(s_n)$	s_{n+1}
1	-6	-22	40	-5.45

Control Systems: Synthetic Division

Example

Iteration No.	s_n	$f(s_n)$	$f'(s_n)$	s_{n+1}
1	-6	-22	40	-5.45
2	-5.45	-3.46	27.7	-5.325

Control Systems: Synthetic Division

Example

Iteration No.	s_n	$f(s_n)$	$f'(s_n)$	s_{n+1}
1	-6	-22	40	-5.45
2	-5.45	-3.46	27.7	-5.325
3	-5.325	-0.0004	25.04	-5.3186

Control Systems: Synthetic Division

Example

Iteration No.	s_n	$f(s_n)$	$f'(s_n)$	s_{n+1}
1	-6	-22	40	-5.45
2	-5.45	-3.46	27.7	-5.325
3	-5.325	-0.0004	25.04	-5.3186
4	-5.3186	≈0		

Hence the first root r_1=-5.3186

The above root can be used to reduce the polynomial order using synthetic division

Control Systems: Synthetic Division

Example

Synthetic Division

1	6	4	2
	-5.3186		
1	0.6814		

x-5.3186

Control Systems: Synthetic Division

Example

Synthetic Division

1	**6**	**4**	**2**
	-5.3186	-3.62409	
1	0.6814	0.3759	

x-5.3186

x-5.3186

Control Systems: Synthetic Division

Example

Synthetic Division

1	**6**	**4**	**2**
	-5.3186	-3.62409	-2
1	0.6814	0.3759	0

x-5.3186

x-5.3186

x-5.3186

Control Systems: Synthetic Division

Example

Synthetic Division

1	6	4	2
	-5.3186	-3.62409	-2
1	0.6814	0.3759	0

Remainder

$$\underbrace{s^2 + 0.6814s + 0.3759}_{\text{Quotient}}$$

Control Systems: Synthetic Division

Example

Solving the quadratic equation

$$s^2 + 0.6814s + 0.3759$$

$$r_2, r_3 = \frac{-0.6814 \pm \sqrt{0.6814^2 - 4 \times 0.3759}}{2}$$
$$= -0.3407 \pm 0.5099j$$

$$(s + 5.3186)(s^2 + 0.6814s + 0.3759)$$
$$= (s + 5.3186)(s + 0.3407 - 0.5099j)(s + 0.3407 + 0.5099j)$$
$$= s^3 + 6s^2 + 4s + 2$$

Control Systems: Synthetic Division

- Synthetic division can be used to easily obtain the roots of any higher degree polynomial and is helpful in the root locus method

- A complex number can also be used as an initial guess

- By using the properties of root-locus, some roots are already known; especially from the root locus arms that are along the real axis. Therefore, by using synthetic division, other roots can be determined

- This method is extensively used in root locus

ME 779 Control Systems

Topic #12

Closed-loop poles on the imaginary axis

Reference textbook:

Control Systems, Dhanesh N. Manik, Cengage Publishing, 2012

Control Systems: Imaginary closed-loop poles

Learning Objectives

- Procedure
- Examples

Control Systems: Imaginary closed-loop poles

Procedure

Replace the denominator of the closed-loop response with $s=j\omega$ and equate the real and imaginary parts to zero

Control Systems: Imaginary closed-loop poles

Example 1

$$G(s)=\frac{K}{s(s+1)}, H(s)=1$$

$$\frac{C(s)}{R(s)}=\frac{G(s)}{1+G(s)H(s)}=\frac{\frac{K}{s(s+1)}}{1+\frac{K}{s(s+1)}}=\frac{K}{s^2+s+K}$$

$$B(s)=s^2+s+K$$

$$\boxed{B(j\omega)=(j\omega)^2+(j\omega)+K=(K-\omega^2)+j\omega}$$

$$\omega=\sqrt{K}$$

$$\omega=0$$

Closed-loop poles are not on the imaginary axis for any positive value of K

Control Systems: Imaginary closed-loop poles

Example 2

$$G(s) = \frac{K}{s(s+2)(S+4)}, H(s) = 1$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\dfrac{K}{s(s+2)(S+4)}}{1+\dfrac{K}{s(s+2)(S+4)}} = \frac{K}{s^3+6s^2+8s+K}$$

Control Systems: Imaginary closed-loop poles

Example 2

$$B(s) = s^3 + 6s^2 + 8s + K$$

$$B(j\omega) = (j\omega)^3 + 6(j\omega)^2 + 8j\omega + K = (K - 6\omega^2) + j(8\omega - \omega^3)$$

$$\omega = \pm\sqrt{8} \qquad K = 6\omega^2 = 48$$

Two closed-loop poles on the imaginary axis for K=48

Control Systems: Imaginary closed-loop poles

Example 3

$$G(s) = \frac{K}{s^2(s+1)} \qquad H(s) = 1$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\dfrac{K}{s^2(s+1)}}{1+\dfrac{K}{s^2(s+1)}} = \frac{K}{s^3+s^2+K}$$

Control Systems: Imaginary closed-loop poles

Example 3

$$B(s) = s^3 + s^2 + K$$

$$\boxed{B(j\omega) = (j\omega)^3 + (j\omega)^2 + K = (K - \omega^2) - j\omega^3}$$

$$\omega = 0 \qquad\qquad K = 0$$

Closed-loop poles are not on the imaginary axis for any positive value of K

Control Systems: Imaginary closed-loop poles

Example 4

$$G(s) = \frac{K}{s^4 + 5s^3 + 8s^2 + 6s} \qquad H(s) = 1$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} = \frac{\dfrac{K}{s^4 + 5s^3 + 8s^2 + 6s}}{1 + \dfrac{K}{s^4 + 5s^3 + 8s^2 + 6s}} = \frac{K}{s^4 + 5s^3 + 8s^2 + 6s + K}$$

Control Systems: Imaginary closed-loop poles

Example 4

$$B(s) = s^4 + 5s^3 + 8s^2 + 6s + K$$

$$B(j\omega) = (j\omega)^4 + 5(j\omega)^3 + 8(j\omega)^2 + 6j\omega + K$$
$$= (\omega^4 - 8\omega^2 + K) + j(6\omega - 5\omega^3)$$

$$\omega = \pm\sqrt{\frac{6}{5}} \qquad K = 8 \times \left(\frac{6}{5}\right) - \left(\frac{6}{5}\right)^2 = 8.16$$

Two closed-loop poles on the imaginary axis for K=8.16

Control Systems: Imaginary closed-loop poles

Example 5

$$G(s)=\frac{K\left(s^2+10s+100\right)}{s^4+20s^3+100s^2+500s+1500}\qquad H(s)=1$$

$$\frac{C(s)}{R(s)}=\frac{G(s)}{1+G(s)H(s)}=\frac{\dfrac{K\left(s^2+10s+100\right)}{s^4+20s^3+100s^2+500s+1500}}{1+\dfrac{K\left(s^2+10s+100\right)}{s^4+20s^3+100s^2+500s+1500}}$$

$$=\frac{K\left(s^2+10s+100\right)}{s^4+20s^3+100s^2+500s+1500+K\left(s^2+10s+100\right)}$$

Control Systems: Imaginary closed-loop poles

Example 5

$$B(s) = s^4 + 20s^3 + 100s^2 + 500s + 1500 + K\left(s^2 + 10s + 100\right)$$

$$= s^4 + 20s^3 + s^2(100 + K) + s(500 + 10K) + 1500 + 100K$$

$$B(j\omega) = (j\omega)^4 + 20(j\omega)^3 + (j\omega)^2(100 + K)$$

$$+ (j\omega)(500 + 10K) + 1500 + 100K$$

$$= \left[\omega^4 - \omega^2(100 + K) + 1500 + 100K\right]$$

$$+ j\left[-20\omega^3 + \omega(500 + 10K)\right]$$

$$\omega = \pm\sqrt{\frac{500 + 10K}{20}}$$

$$K^2 - 200K + 1500 = 0$$

K_1= 7.8 and K_2 = 192.

Four closed-loop poles on the imaginary axis for K_1= 7.8 and K_2 = 192

Control Systems: Imaginary closed-loop poles

Example 6 $G(s)=\dfrac{K}{s^4+s^3+s^2+s}$ $H(s)=1$

$$\frac{C(s)}{R(s)}=\frac{G(s)}{1+G(s)H(s)}$$

$$=\frac{\dfrac{K}{s^4+s^3+s^2+s}}{1+\dfrac{K}{s^4+s^3+s^2+s}}$$

$$=\frac{K}{s^4+s^3+s^2+s+K}$$

Control Systems: Imaginary closed-loop poles

Example 6

$$B(s) = s^4 + s^3 + s^2 + s + K$$

$$B(j\omega) = (j\omega)^4 + (j\omega)^3 + (j\omega)^2 + j\omega + K = (\omega^4 - \omega^2 + K) + j(\omega - \omega^3)$$

$$\omega = \pm 1 \qquad\qquad K = 0$$

No closed-loop poles on the imaginary axis for K>0

ME 779 Control Systems

Topic #13

Routh-Hurwitz's Stability Criterion

Reference textbook:

Control Systems, Dhanesh N. Manik, Cengage Publishing, 2012

Control Systems: Routh-Hurwitz's stability criterion

$$B(s) = a_n s^n + a_{n-1} s^{n-1} + \cdots a_1 s + a_0$$

Characteristic equation

$$(s - r_1)(s - r_2)\cdots(s - r_n) = 0$$

Roots of the characteristic equation

$$r_i, i = 1, 2...$$

$$B(s) = s^n - (r_1 + r_2 + \cdots r_n) s^{n-1}$$

$$+ (r_1 r_2 + r_2 r_3 + r_1 r_3 + \cdots) s^{n-2}$$

$$- (r_1 r_2 r_3 + r_1 r_2 r_4 + \cdots) s^{n-3} + \cdots$$

$$(-1)^n r_1 r_2 r_3 \ldots r_n = 0$$

Control Systems: Routh-Hurwitz's stability criterion

$$\begin{aligned}B(s) = s^n &- (sum\, of\ all\, the\, roots)s^{n-1} \\ &+ (sum\, of\ the\ products\, of\ the\, roots\, taken\ 2\ at\, a\, time)s^{n-2} \\ &- (sum\, of\ the\ products\, of\ the\, roots\, taken\ 3\ at\, a\, time)s^{n-3} + \cdots \\ &+ (-1)^n (product\, of\ all\, n\, roots) = 0\end{aligned}$$

Control Systems: Routh-Hurwitz's stability criterion

Necessary condition: coefficients of the characteristic polynomial must be positive

Example 1

Consider a third order polynomial

$$B(s) = s^3 + 3s^2 + 16s + 130$$

Although the coefficients of the above polynomial are positive, determine the roots and hence prove that the rule about coefficients being positive is only a necessary condition for the roots to be in the left s-plane.

Example 1

$$r_1 = -5; r_{2,3} = 1 \pm 5j$$

By using Newton-Raphson's method

Therefore, from the above example, the condition that coefficients of a polynomial should be positive for all its roots to be in the left s-plane is only a necessary condition

Control Systems: Routh-Hurwitz's stability criterion

Sufficient condition

Method I (using determinants)

$$\Delta_n = \begin{vmatrix} a_{n-1} & a_{n-3} & a_{n-5} & \cdots \\ a_n & a_{n-2} & a_{n-4} & \cdots \\ 0 & a_{n-1} & a_{n-3} & \cdots \\ & & & \end{vmatrix}$$

decreases by two along the row
increases by one down the column

Sufficient condition

Method I (using determinants)

$$\Delta_1 = a_{n-1} > 0, \Delta_2 = \begin{vmatrix} a_{n-1} & a_{n-3} \\ a_n & a_{n-2} \end{vmatrix} > 0, \Delta_3 = \begin{vmatrix} a_{n-1} & a_{n-3} & a_{n-5} \\ a_n & a_{n-2} & a_{n-4} \\ 0 & a_{n-1} & a_{n-3} \end{vmatrix} > 0 \cdots$$

Sufficient condition

Method II Using array

$$
\begin{array}{c|ccc}
s^n & a_n & a_{n-2} & a_{n-4} \\
s^{n-1} & a_{n-1} & a_{n-3} & a_{n-5} \\
s^{n-2} & b_{n-1} & b_{n-3} & b_{n-5} \\
s^{n-3} & c_{n-1} & c_{n-3} & c_{n-5} \\
\vdots & & &
\end{array}
$$

$$b_{n-1} = \frac{(a_{n-1})(a_{n-2}) - a_n(a_{n-3})}{a_{n-1}}$$

$$b_{n-3} = \frac{(a_{n-1})(a_{n-4}) - a_n(a_{n-5})}{a_{n-1}}$$

$$c_{n-1} = \frac{(b_{n-1})(a_{n-3}) - a_{n-1}(b_{n-3})}{b_{n-1}}$$

Control Systems: Routh-Hurwitz's stability criterion

Sufficient condition

Method II Using array

number of roots of B(s) with positive real parts is equal to the number of sign changes a_n, a_{n-1}, b_{n-1}, c_{n-1}, etc.

Example 2

$$G(s)=\frac{K}{s(s+1)} \qquad H(s)=1$$

$$\frac{C(s)}{R(s)}=\frac{G(s)}{1+G(s)H(s)}=\frac{\dfrac{K}{s(s+1)}}{1+\dfrac{K}{s(s+1)}}=\frac{K}{s^2+s+K}$$

Control Systems: Routh-Hurwitz's stability criterion

Example 2

Method I using determinants

$$B(s) = s^2 + s + K$$

$$B(s) = a_2 s^2 + a_1 s + a_0$$

$$\Delta_2 = \begin{vmatrix} 1 & 0 \\ 1 & K \end{vmatrix}$$

$\Delta_1 = 1 > 0$

$\Delta_2 = K$

The system is always stable for K>0

Control Systems: Routh-Hurwitz's stability criterion

Example 2 **Method II using array**

$$B(s)=s^2+s+K$$

$$B(s)=a_2s^2+a_1s+a_0$$

$$\begin{array}{c|cc} s^n & 1 & K \\ s^{n-1} & 1 & 0 \\ s^{n-2} & K & \end{array}$$

There are no sign changes

The system is always stable for K>0

Example 3

$$G(s) = \frac{K}{s(s+2)(S+4)} \qquad H(s) = 1$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\dfrac{K}{s(s+2)(S+4)}}{1+\dfrac{K}{s(s+2)(S+4)}} = \frac{K}{s^3+6s^2+8s+K}$$

Control Systems: Routh-Hurwitz's stability criterion

Example 3

Method I using determinants

$$B(s) = s^3 + 6s^2 + 8s + K$$

$$B(s) = a_3 s^3 + a_2 s^2 + a_1 s + a_0$$

$$\Delta_3 = \begin{vmatrix} a_2 & a_0 & 0 \\ a_3 & a_1 & 0 \\ 0 & a_2 & a_0 \end{vmatrix} \qquad \Delta_3 = \begin{vmatrix} 6 & K & 0 \\ 1 & 8 & 0 \\ 0 & 6 & K \end{vmatrix}$$

Δ_1=6 >0

Δ_2=48-K>0

Δ_3=K(48-K)>0

Sufficient conditions for and Stability

The feedback system is stable for values of K<48

Control Systems: Routh-Hurwitz's stability criterion

Example 3

Method II using array

$$B(s) = s^3 + 6s^2 + 8s + K$$

$$B(s) = a_3 s^3 + a_2 s^2 + a_1 s + a_0$$

$$\begin{array}{c|cc} s^n & a_3 & a_1 \\ s^{n-1} & a_2 & a_0 \\ s^{n-2} & \dfrac{a_2 a_1 - a_3 a_0}{a_2} & 0 \end{array} \qquad \begin{array}{c|cc} s^n & 1 & 8 \\ s^{n-1} & 6 & K \\ s^{n-2} & \dfrac{48 - K}{6} & 0 \end{array}$$

All values positive for K<48

Example 4

$$G(s) = \frac{K}{s^2(s+1)} \qquad H(s) = 1$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\dfrac{K}{s^2(s+1)}}{1+\dfrac{K}{s^2(s+1)}} = \frac{K}{s^3+s^2+K}$$

Example 4

Method I using determinants

$$B(s) = s^3 + s^2 + K$$

$$B(s) = a_3 s^3 + a_2 s^2 + a_1 s + a_0$$

$$\Delta_2 = \begin{vmatrix} a_2 & a_0 \\ a_3 & a_1 \end{vmatrix} = \begin{vmatrix} 1 & K \\ 1 & 0 \end{vmatrix}$$

$\Delta_1 = 1 > 0$

$\Delta_2 = -K$

Always negative

The system is always unstable

Example 4 | Method II using array

$$B(s) = s^3 + s^2 + K$$

$$B(s) = a_3 s^3 + a_2 s^2 + a_1 s + a_0$$

$$\begin{array}{c|cc} s^n & a_3 & a_1 \\ s^{n-1} & a_2 & a_0 \\ s^{n-2} & \dfrac{a_2 a_1 - a_3 a_0}{a_2} & 0 \end{array}$$

$$\boxed{\begin{array}{c|cc} s^n & 1 & 0 \\ s^{n-1} & 1 & K \\ s^{n-2} & -K & 0 \end{array}}$$

The system is always unstable