# ME 779 Control Systems

## Topic #28

## Nyquist plots:
## Gain and phase margin

**Reference textbook:**

**Control Systems, Dhanesh N. Manik,**
**Cengage Publishing, 2012**

# Nyquist plots: Gain and Phase margin

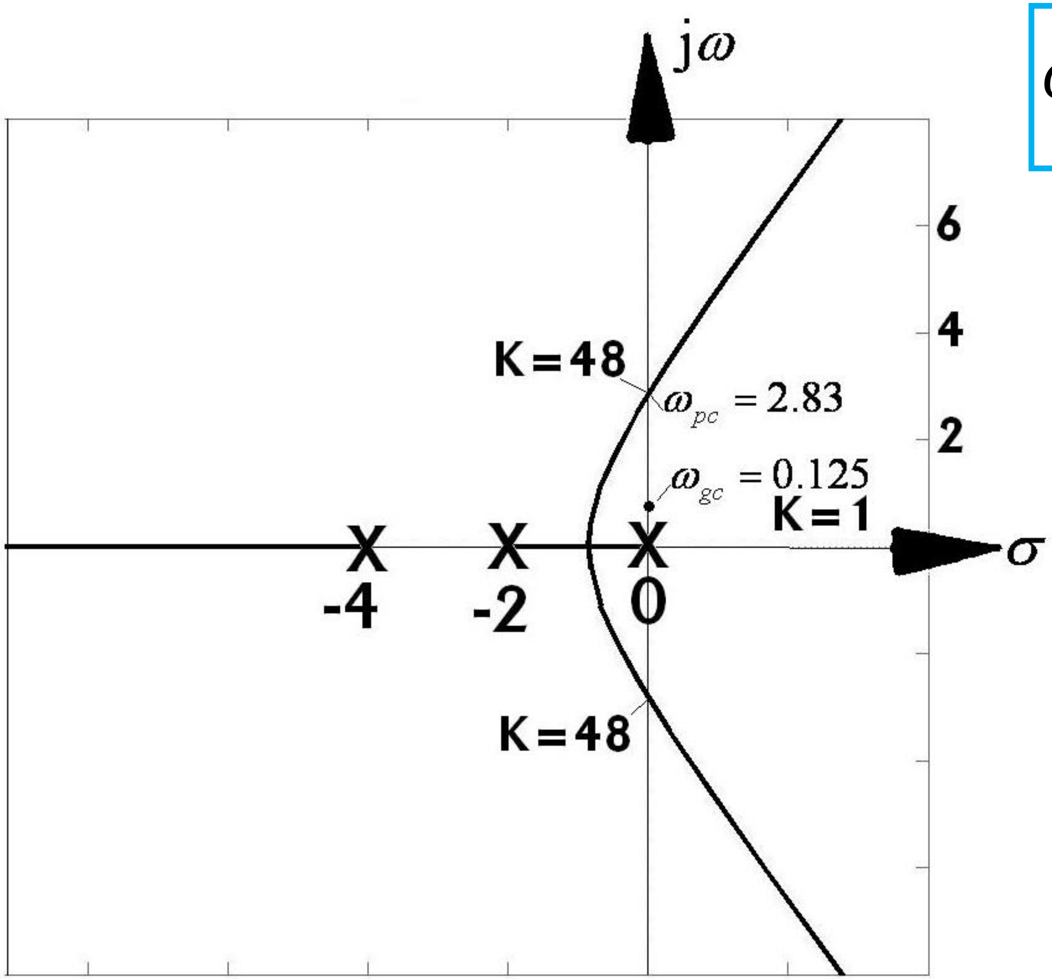
**Gain Margin and Phase Margin**

**phase crossover frequency $\omega_p$ is the frequency at which the open-loop transfer function has a phase of 180º**

**The gain crossover frequency $\omega_g$ is the frequency at which the open-loop transfer function has a unit gain**

# Nyquist plots: Gain and Phase margin

$$G(s)H(s) = \frac{K}{s(s+2)(s+4)}$$



# Nyquist plots: Gain and Phase margin

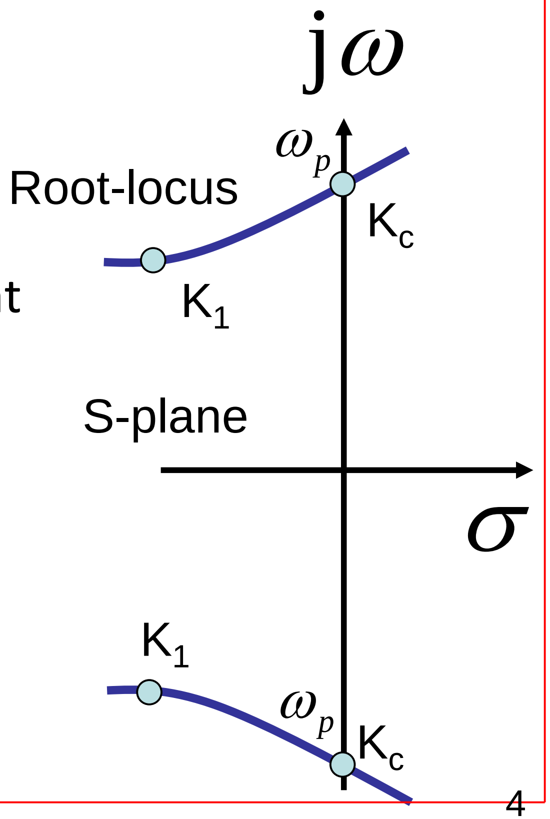
Beginning from the gain margin equation

based on root-locus plots $GM = 20\log\frac{K_c}{K_1}$,

where $K_c$ is the open-loop gain corresponding to marginal stability and $K_1$ is the open-loop gain at another arbitrary point on the root-locus, prove that

$$GM = -20\log\left|G(j\omega_p)H(j\omega_p)\right|;$$

$\omega_p$ is the phase crossover frequency.



# Nyquist plots: Gain and Phase margin

The open-loop transfer function in terms of open-loop poles and zeros is given by

$$G(s)H(s) = \frac{K(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$$

$$\left|G(j\omega)H(j\omega)\right| = \frac{K\left|(j\omega-z_1)(j\omega-z_2)\cdots(j\omega-z_m)\right|}{\left|(j\omega-p_1)(j\omega-p_2)\cdots(j\omega-p_n)\right|}$$

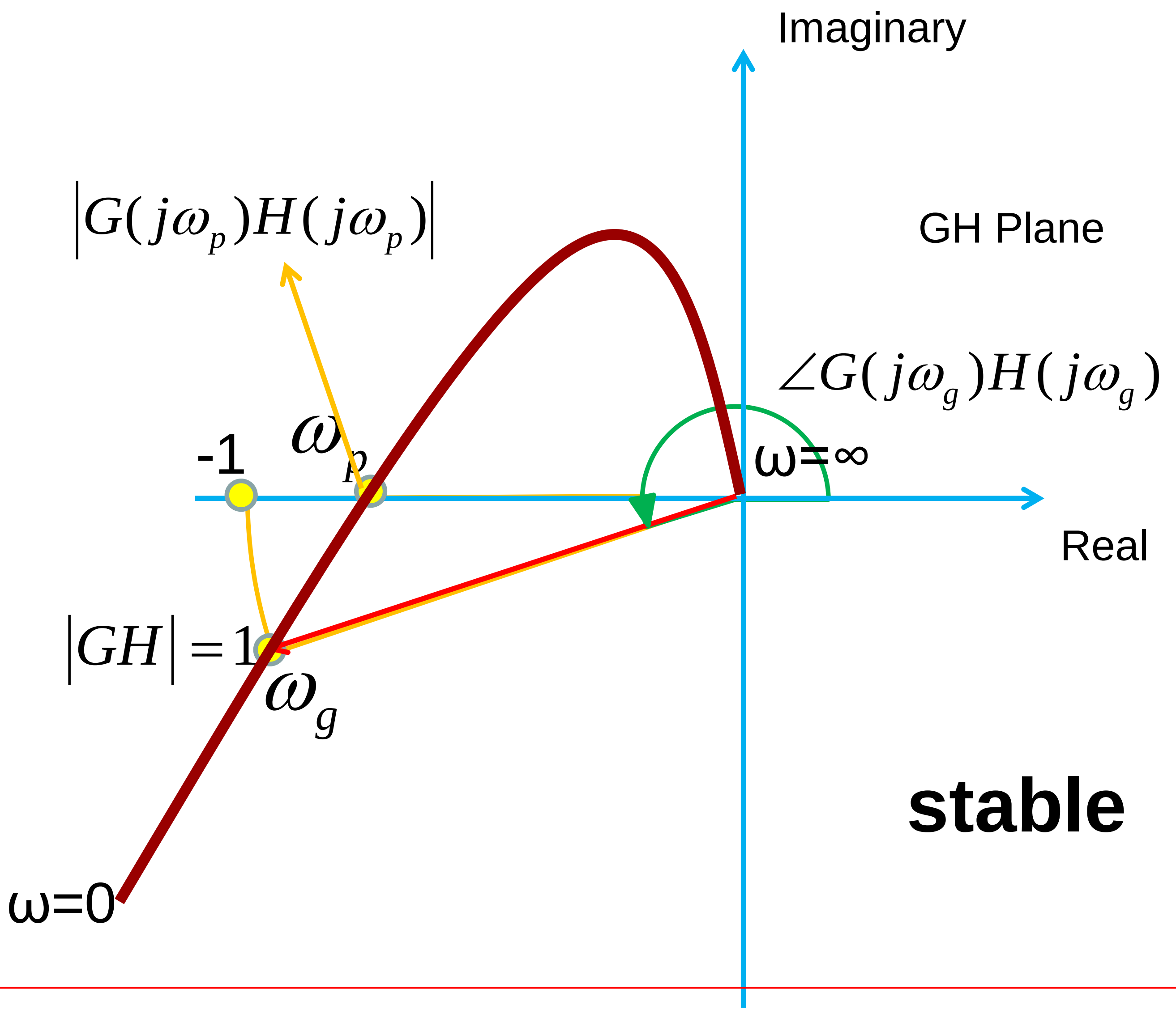
Magnitude of the Open-loop frequency response function

$$\left|G(j\omega_p)H(j\omega_p)^{K_1}\right| = \frac{K_1\left|(j\omega_p-z_1)(j\omega_p-z_2)\cdots(j\omega_p-z_m)\right|}{\left|(j\omega_p-p_1)(j\omega_p-p_2)\cdots(j\omega_p-p_n)\right|}$$

$$\left|G(j\omega_p)H(j\omega_p)^{K_c}\right| = 1 = \frac{K_c\left|(j\omega_p-z_1)(j\omega_p-z_2)\cdots(j\omega_p-z_m)\right|}{\left|(j\omega_p-p_1)(j\omega_p-p_2)\cdots(j\omega_p-p_n)\right|}$$

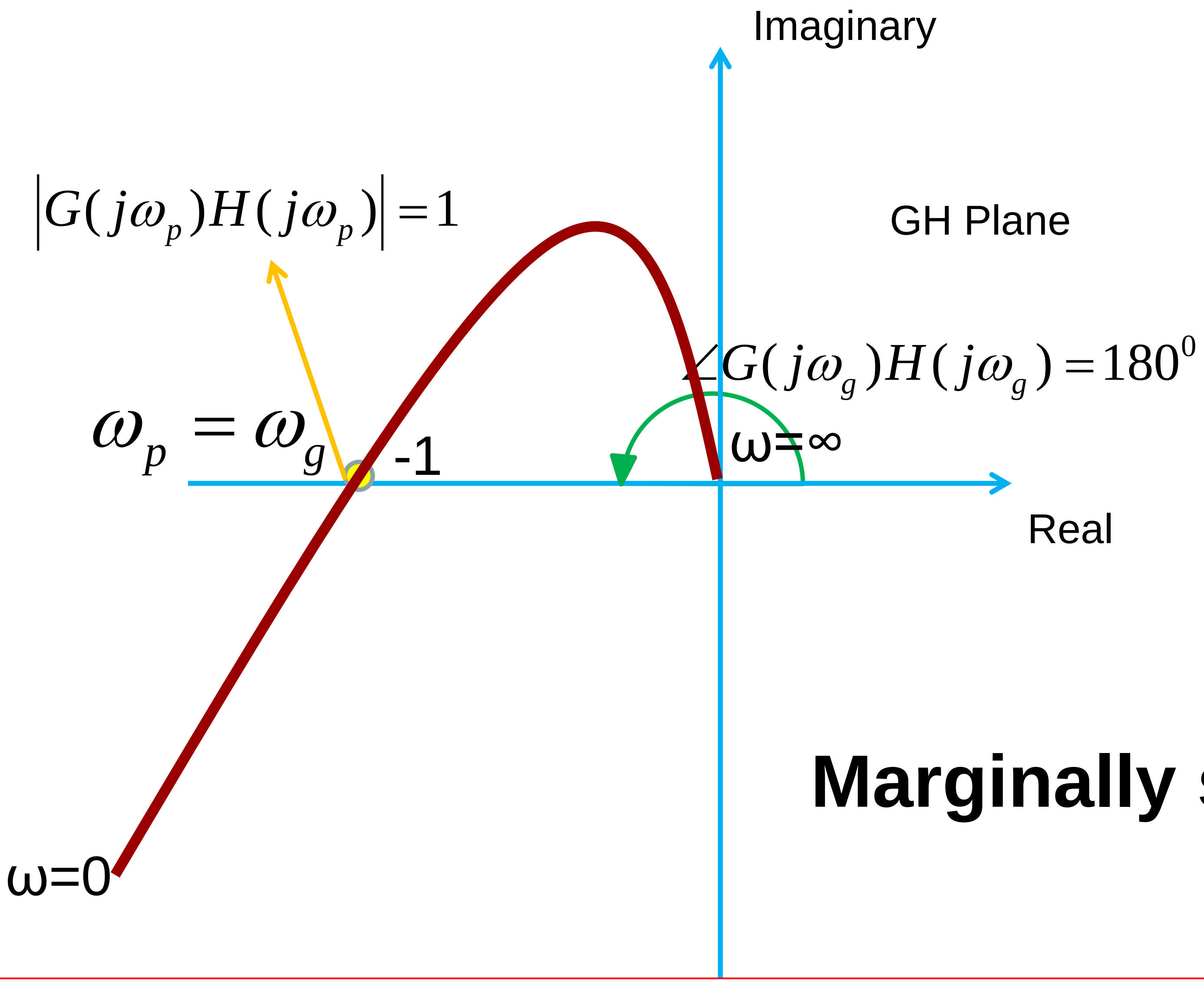
The ratio of equations result in $GM = -20\log\left|G(j\omega_p)H(j\omega_p)\right|$

# Nyquist plots: Gain and Phase margin



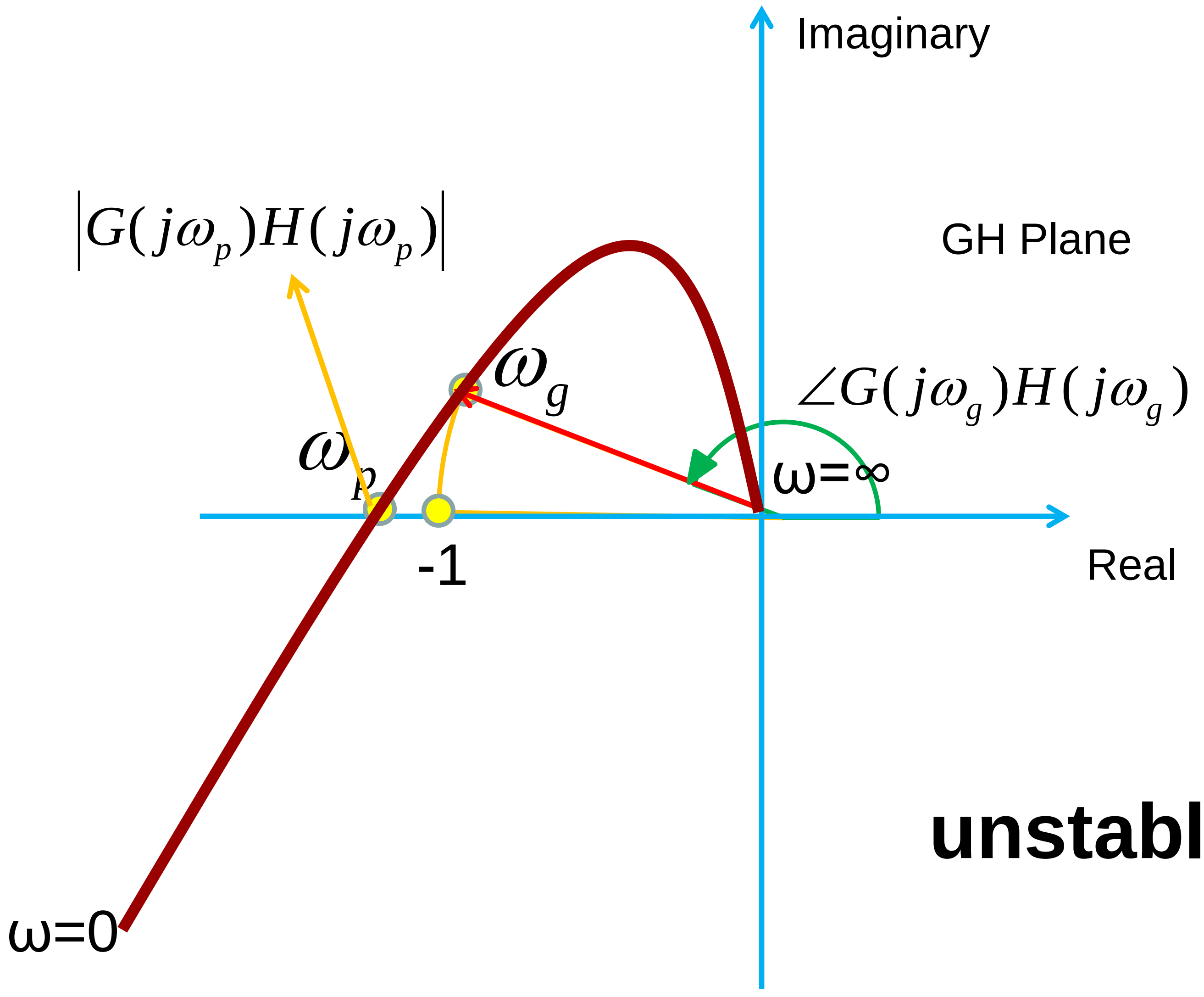

**stable**

# Nyquist plots: Gain and Phase margin




**Marginally stable**

# Nyquist plots: Gain and Phase margin




**unstable**

# Nyquist plots: Gain and Phase margin

**Gain Margin and Phase Margin**

Gain margin

$$M = -20\log\left|G(j\omega_p)H(j\omega_p)\right|$$

Phase margin

$$\gamma = \angle G(j\omega_g)H(j\omega_g) - 180^o$$

# Nyquist plots: Gain and Phase margin

## Example 1

**Determine gain margin, phase margin and stability of the feedback system whose open-loop transfer function given by**

$$G(s)H(s) = \frac{K}{s(s+1)}$$

# Nyquist plots: Gain and Phase margin

## Example 1

| No. | Frequency, rad/s | Magnitude | Phase, degrees |
|---|---|---|---|
| 1 | 0 | ∞ | 270 |
| 2 | 0.2 | 4.9029 | 259 |
| 3 | 0.4 | 2.3212 | 248 |
| 4 | 0.786 | 1 | 232 |
| 5 | 0.8 | 0.9761 | 231 |
| 6 | 1 | 0.7071 | 225 |
| 7 | 4 | 0.0606 | 194 |
| 8 | 10 | 0.01 | 186 |
| 9 | 50 | 0.0004 | 181 |
| 10 | 100 | 0.0001 | 181 |
| 11 | 200 | ≈0 | ≈180 |

# Nyquist plots: Gain and Phase margin

## Example 1

Phase crossover frequency $\omega_p$ is very large (infinity). Gain at $\omega_p$ =0 and hence gain margin=

$$-20\log\left|G(j\omega_p)H(j\omega_p)\right| = \infty$$

Gain crossover frequency $\omega_g$ = 0.786 rad/s

Phase angle corresponding to $\omega_g$ =232$^o$

Phase margin is 232$^0$ -180$^0$=52$^o$

The system is stable for the current value of K=1 and will become unstable when the value of K is increased infinite times. That means that the system is always stable for all values of K

# Nyquist plots: Gain and Phase margin

## Example 2

**Determine gain margin, phase margin and stability of the feedback system whose open-loop transfer function given by**

$$G(s)H(s) = \frac{55}{s(s+2)(s+4)}$$

# Nyquist plots: Gain and Phase margin

**Example 2**

| No. | Frequency | Magnitude | Phase, degrees |
|---|---|---|---|
| 1 | 1.5 | 3.4332 | 213 |
| 2 | 2 | 2.1741 | 198 |
| 3 | 2.5 | 1.4568 | 187 |
| 4 | 2.83 | 1.1446 | 180 |
| 5 | 3 | 1.017 | 177 |
| 6 | 3.5 | 0.7334 | 169 |
| 7 | 4.5 | 0.4122 | 156 |
| 8 | 5 | 0.319 | 150 |
| 9 | 5.5 | 0.2513 | 146 |
| 10 | 6 | 0.201 | 142 |
| 11 | 7 | 0.1339 | 136 |
| 12 | 8 | 0.0932 | 131 |
| 13 | 9 | 0.0673 | 126 |

**Magnitude and phase of the open-loop frequency transfer function (K=55)**

# Nyquist plots: Gain and Phase margin

**Example 2**

**Phase crossover frequency 2.83 rad/s**

$$K^* = 55/1.1446 = 48$$

The gain at which the system becomes marginally stable

Gain margin

$$M = -20\log\left|G(j\omega_p)H(j\omega_p)\right|$$

$$= -20\log\left|1.1446\right| = -1.17\,\text{dB}$$

# Nyquist plots: Gain and Phase margin

## Example 2

Gain crossover frequency =3 rad/s and the corresponding angle Of GH=177$^o$

Phase margin=177-180=-3$^o$

The system is unstable for K=55

# Nyquist plots: Gain and Phase margin

## Example 3

**Determine gain margin, phase margin and stability of the feedback system whose open-loop transfer function given by**

$$G(s)H(s) = \frac{K}{s^2(s+1)}$$

# Nyquist plots: Gain and Phase margin

**Example 3**

| **No.** | **Frequency, rad/s** | **Magnitude** | **Phase, degrees** |
|---|---|---|---|
| **1** | 0 | ∞ | 180 |
| **2** | 0.4 | 5.803 | 158 |
| **4** | 0.5 | 3.5777 | 153 |
| **5** | 0.8 | 1.2201 | 141 |
| **6** | 0.87 | 1 | 139 |
| **7** | 1 | 0.7071 | 135 |
| **8** | 2 | 0.1118 | 117 |
| **9** | 3 | 0.0351 | 108 |
| 10 | 4 | 0.0152 | 104 |
| 11 | 5 | 0.0078 | 101 |

# Nyquist plots: Gain and Phase margin

## Example 3

The phase crossover frequency is 0 rad/s and the corresponding magnitude is infinity

$$M = -20\log\left|G(j\omega_p)H(j\omega_p)\right|$$

$$= -20\log\left|\infty\right| = -\infty\,\text{dB}$$

# Nyquist plots: Gain and Phase margin

The gain crossover frequency is 0.87 rad/s and the corresponding phase is $139^0$

Phase margin $=139^0 - 180^0 = -41^0$

The system is unstable for K=1. Since the gain margin is negative infinity, open-loop gain K has to be decreased infinite times for the system to be stable. Hence this system is unstable for all values of K

# Nyquist plots: Gain and Phase margin

**Conclusion**