ME 779 Control Systems

Topic #29

Nyquist plots: Closed-loop response

Reference textbook:

Control Systems, Dhanesh N. Manik, Cengage Publishing, 2012

Nyquist plots: Closed-loop response

CLOSED LOOP FREQUENCY RESPONSE

$$\frac{C(j\omega)}{R(j\omega)} = \frac{G(j\omega)}{1+G(j\omega)H(j\omega)}$$

Nyquist plots: Closed-loop response

Peak Magnitude

$$M_r = 20\log\left|\frac{C(j\omega)}{R(j\omega)}\right|\text{dB}$$

3 dB is considered good

Nyquist plots: Closed-loop response

Constant M-circles for unity feedback systems

$$M(j\omega) = \frac{G(j\omega)}{1+G(j\omega)}$$

$$G(j\omega) = x + jy$$

$$\left|M(j\omega)\right| = \frac{\sqrt{x^2 + y^2}}{\sqrt{(1+x)^2 + y^2}}$$

Nyquist plots: Closed-loop response

Constant M-circles for unity feedback systems

$$M^2(1+x)^2 + M^2 y^2 = x^2 + y^2$$

$$x^2(1-M^2) + (1-M^2)y^2 - 2M^2 x = M^2$$

$$x^2 + y^2 - 2\frac{M^2}{1-M^2}x = \frac{M^2}{1-M^2}$$

Nyquist plots: Closed-loop response

Constant M-circles for unity feedback systems

$$\left(x-\frac{M^2}{1-M^2}\right)^2+y^2=\left(\frac{M}{1-M^2}\right)^2$$

Adding $\left(\frac{M^2}{1-M^2}\right)^2$

The above equation represents a family of circles with its center at $\left(\frac{M^2}{1-M^2},0\right)$ **and radius** $\left|\frac{M}{1-M^2}\right|$.

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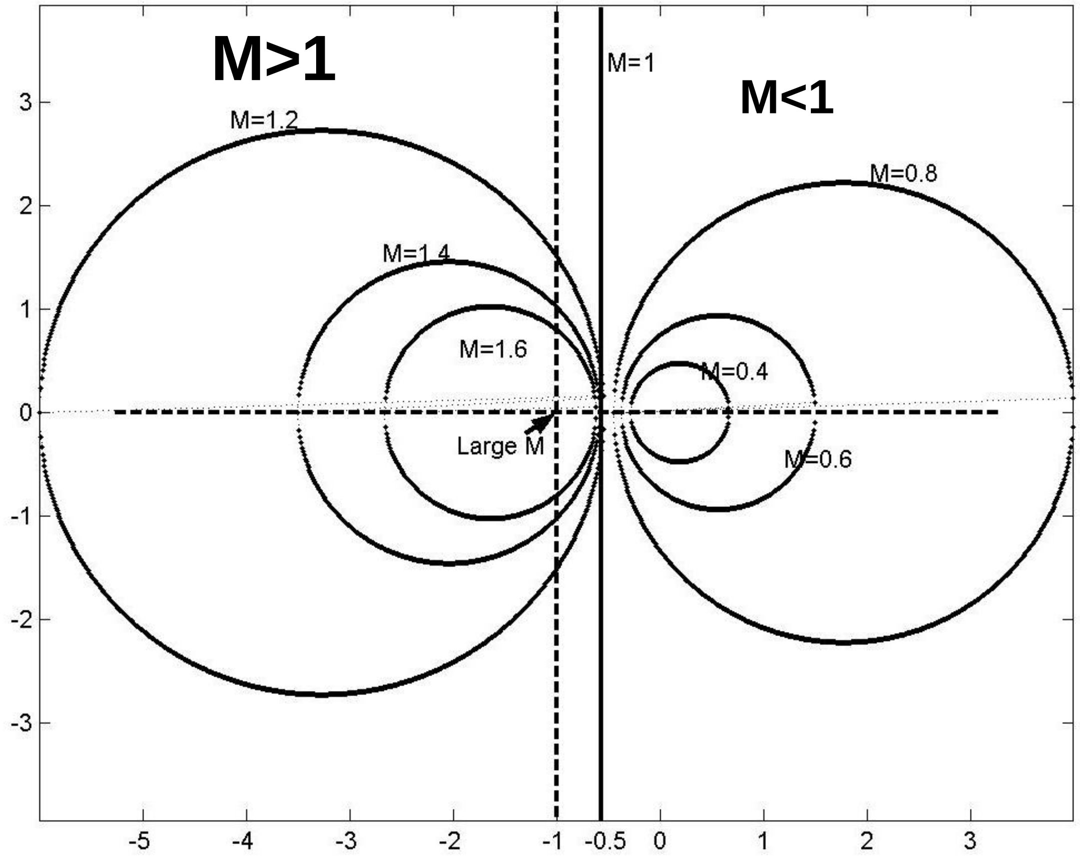
Constant M-circles for unity feedback systems



Family of M-circles corresponding to the close loop magnitudes (M) of a unit feedback system

Nyquist plots: Closed-loop response

Constant N-circles

$$\angle M = \alpha = \frac{\angle G(j\omega)}{\angle 1 + G(j\omega)}$$

$$\alpha = \tan^{-1}\frac{y}{x} - \tan^{-1}\frac{y}{1+x}$$

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Constant N-circles

A **B**

$$N = \tan\left(\tan^{-1}\frac{y}{x} - \tan^{-1}\frac{y}{1+x} \right)$$

tan(α)=N

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$N = \frac{\dfrac{y}{x} - \dfrac{y}{1+x}}{1 + \left(\dfrac{y}{x}\right)\left(\dfrac{y}{1+x}\right)}$$

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Constant N-circles

$$N = \frac{y}{x^2 + x + y^2} \quad \Rightarrow \quad x^2 + x + y^2 - \frac{y}{N} = 0$$

Add $\frac{1}{4} + \frac{1}{4N^2}$ **on both sides**

$$x^2 + x + y^2 - \frac{y}{N} + \frac{1}{4} + \frac{1}{4N^2} = \frac{1}{4} + \frac{1}{4N^2}$$

$$\left(x + \frac{1}{2}\right)^2 + \left(y - \frac{1}{2N}\right)^2 = \frac{1}{4} + \left(\frac{1}{2N}\right)^2$$

Nyquist plots: Closed-loop response

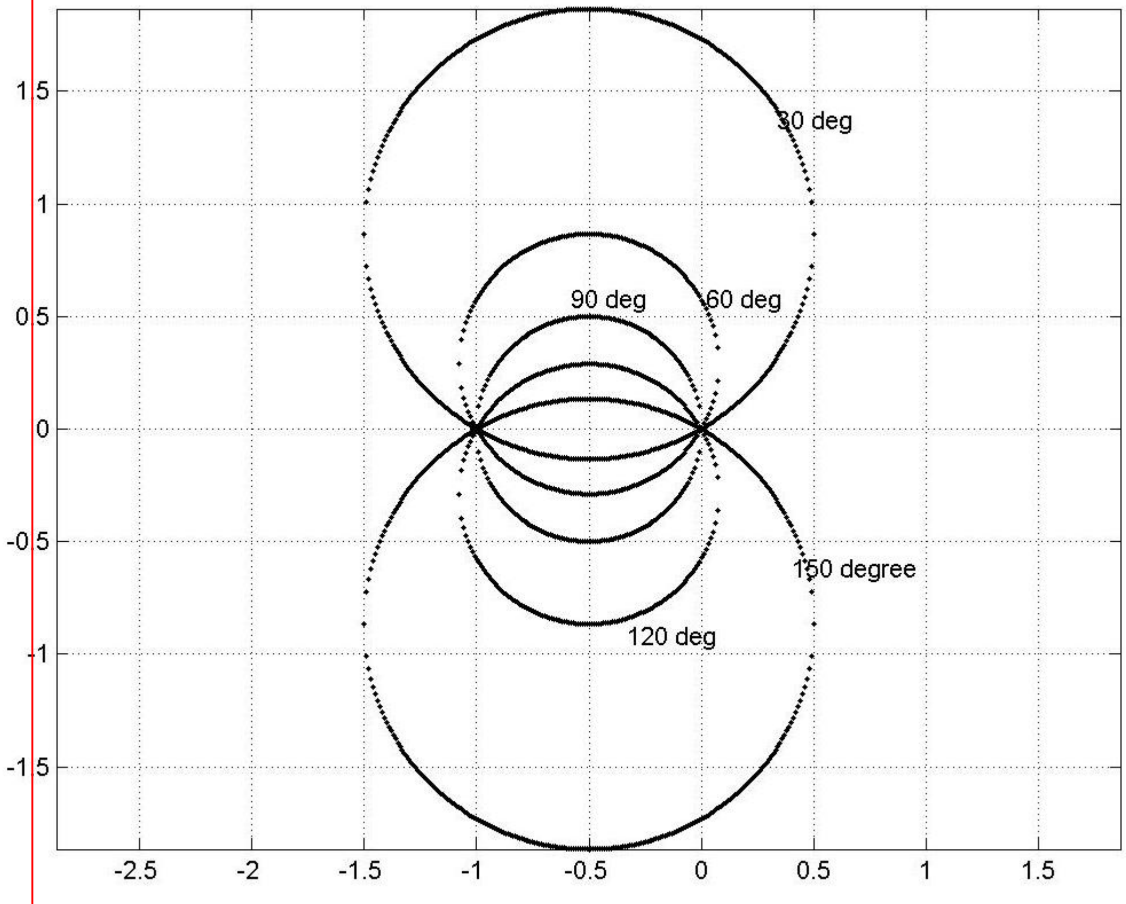
Constant N-circles

$$\left(x+\frac{1}{2}\right)^2+\left(y-\frac{1}{2N}\right)^2=\frac{1}{4}+\left(\frac{1}{2N}\right)^2$$

The above equation represents a family of circles with its center at $\left(-\frac{1}{2},\frac{1}{2N}\right)$ and radius $\sqrt{\frac{1}{4}+\left(\frac{1}{2N}\right)^2}$

Nyquist plots: Closed-loop response

Constant N-circles



Nyquist plots: Closed-loop response

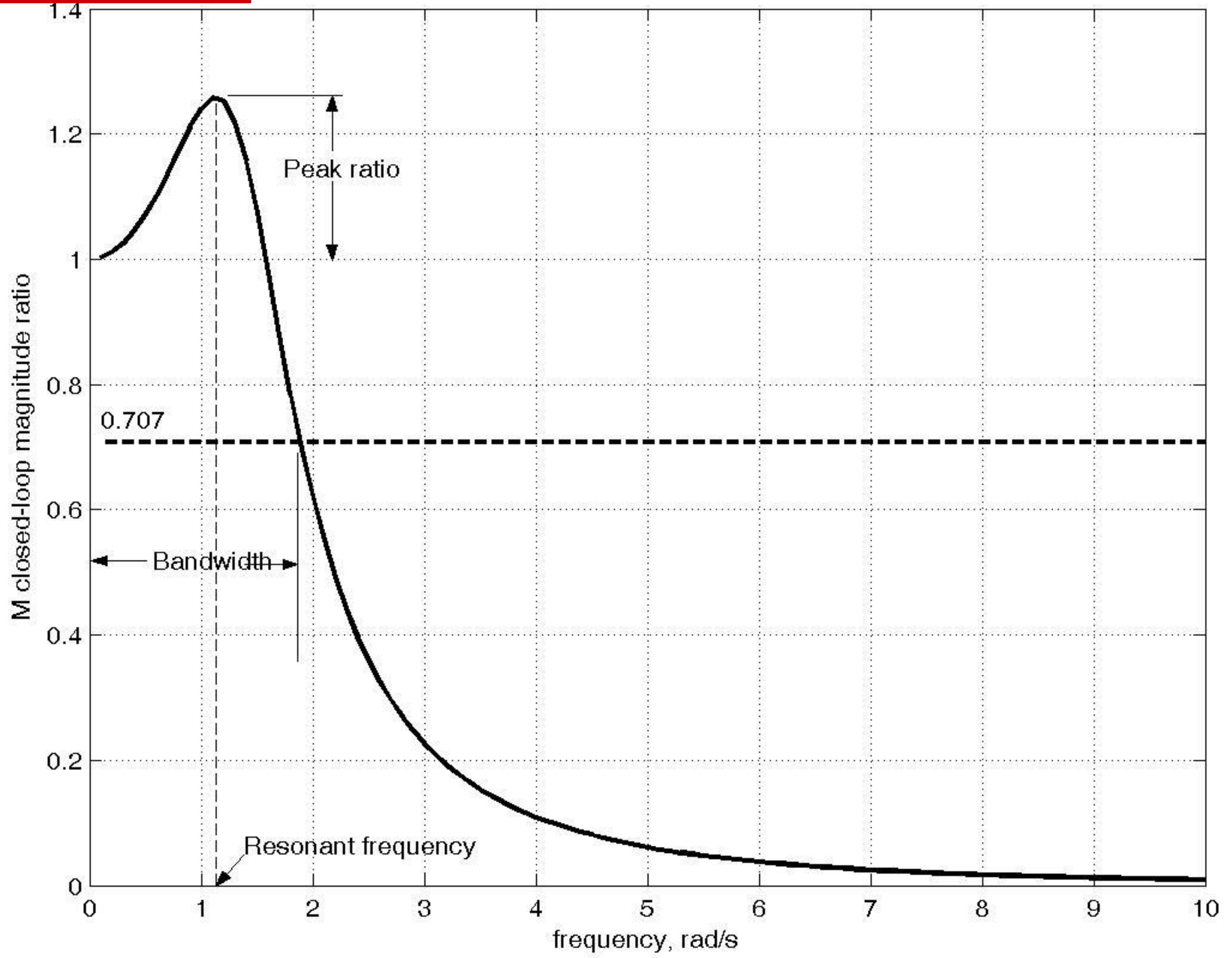
Example

Determine the closed-loop magnitude ratio and bandwidth of the feedback system whose forward transfer function is given by $G(s) = \frac{10}{s(s+2)(s+4)}$ **and H(s)=1, by (1) direct computation and (2) using M and N circles**

$$G(s) = \frac{10}{s(s+2)(s+4)}$$

Nyquist plots: Closed-loop response

Example



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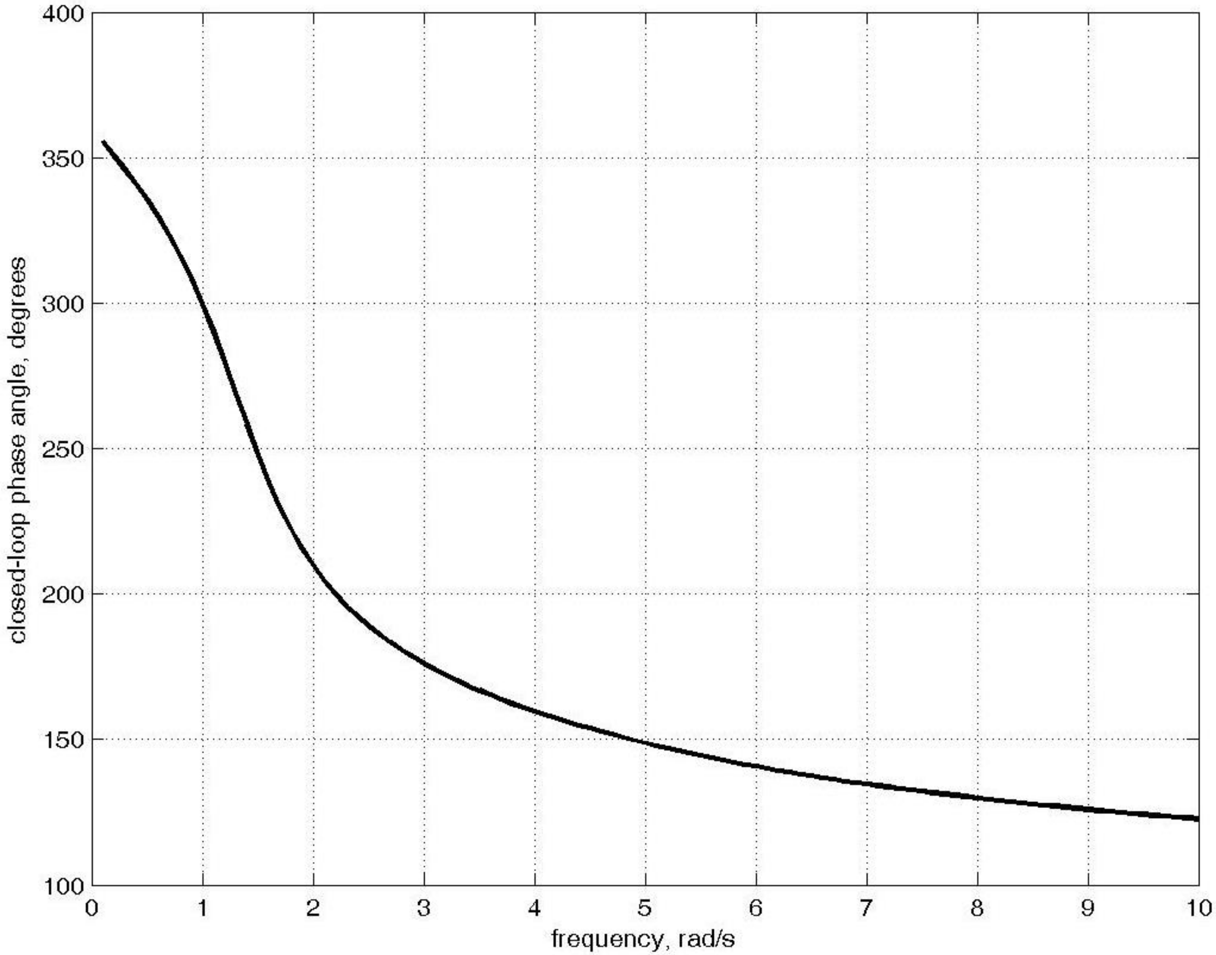
Example



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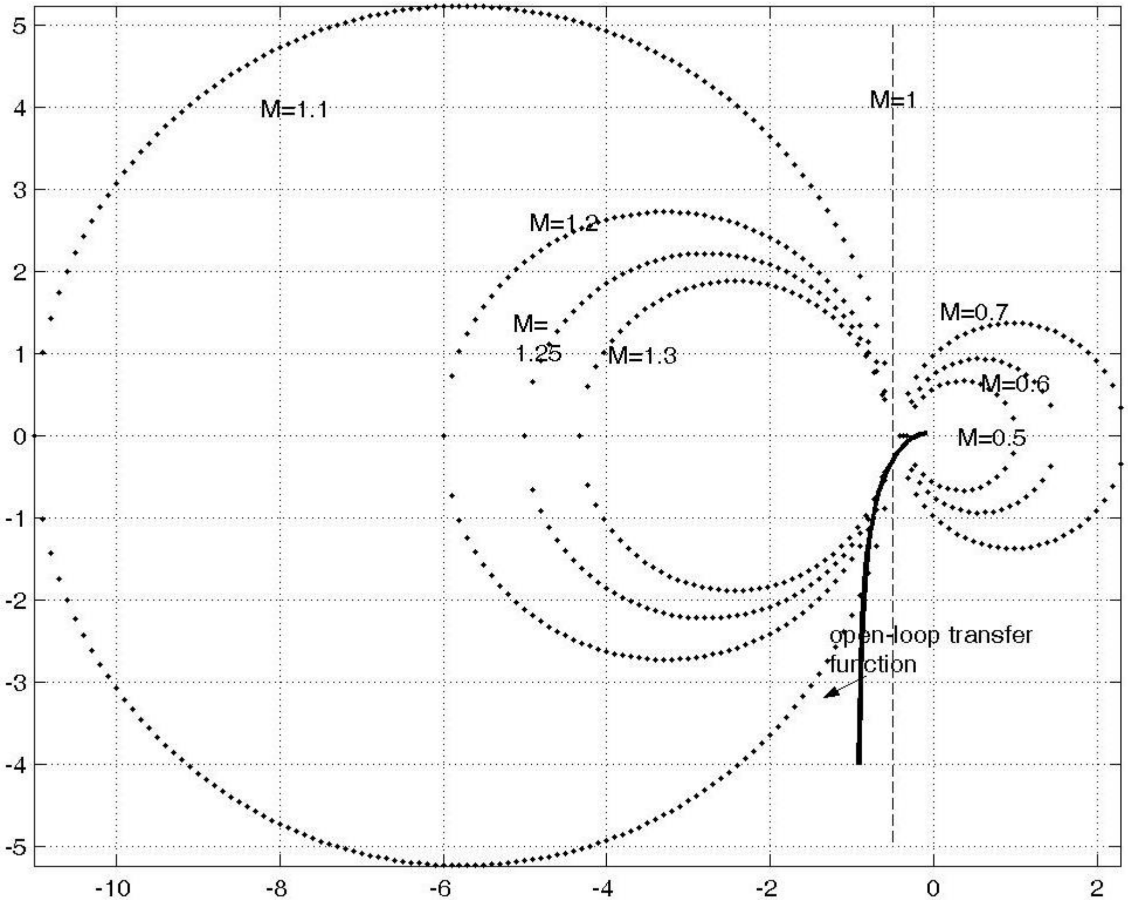
Closed-loop magnitude and phase

Freq., rad/s	Closed-loop Magnitude ratio	Closed-loop Phase angle, deg
0.1	1.0	355
0.5	1.1	335
0.8	1.2	316
0.9	1.2	308
1.0	1.2	300
1.1	1.3	290

Freq., rad/s	Closed-loop Magnitude ratio	Closed-loop Phase angle, deg
1.2	1.3	280
1.3	1.2	269
1.4	1.2	258
1.5	1.1	248
1.6	1.0	238
1.7	0.9	230
1.8	0.8	222
1.9	0.7	216
2.0	0.6	210

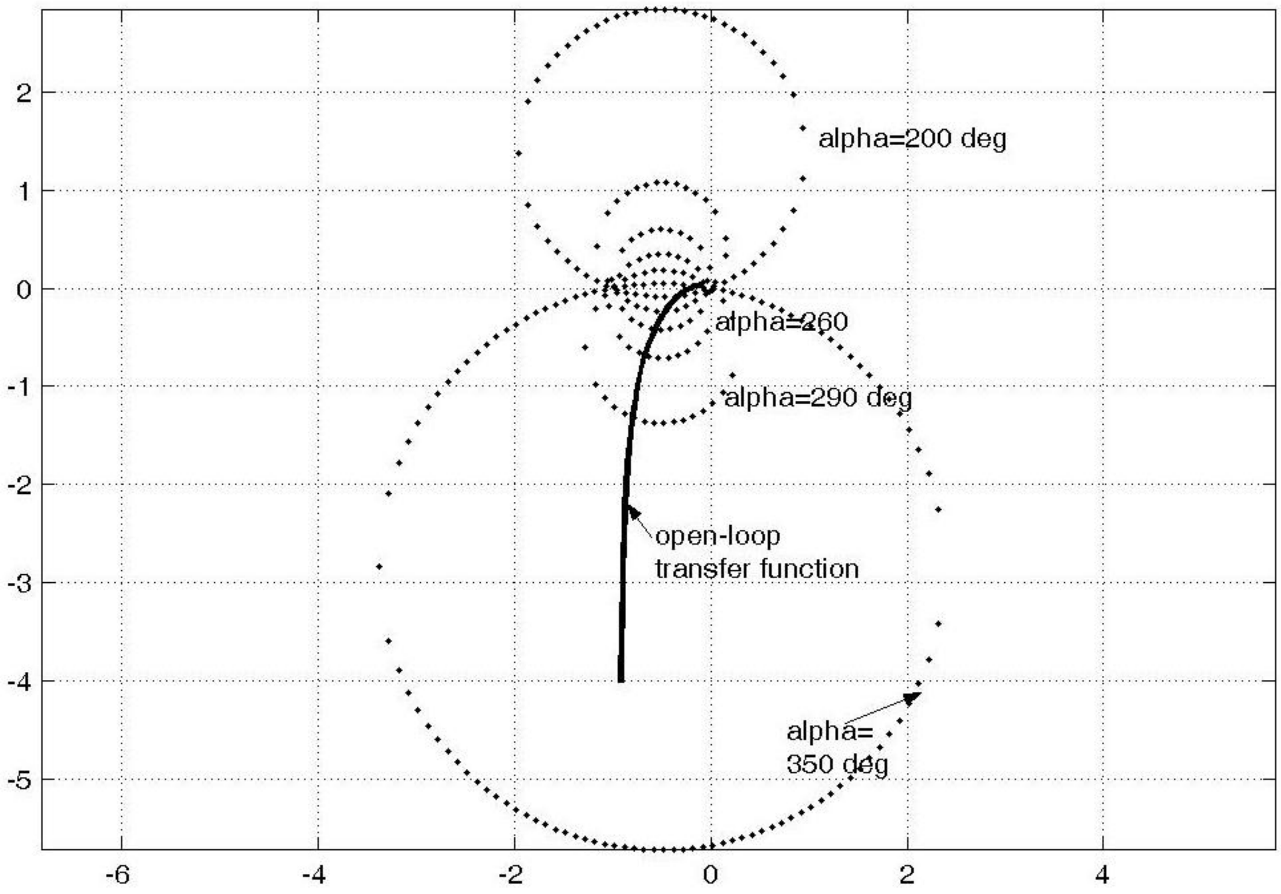
Nyquist plots: Closed-loop response

Example



Nyquist plots: Closed-loop response

Example



Nyquist plots: Closed-loop response

Conclusion