Introduction of Renewable Energy Technologies

Lecture-08

Solar Angle and Estimation of Solar Radiation

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Recap of the last lecture

- Apparent motion of the Sun
- Declination angle between equatorial plane and plane of revolution
- Motion of the Sun at given latitude throughout the year
- Optimum angle of solar collector

In this lecture

- Solar radiation on collector surface
- Local apparent time
- Sunrise, sunset time and day length
- Estimation of Solar radiation at any given time and at any surface

Solar Radiation on Collector Surface

Solar radiation at the collector

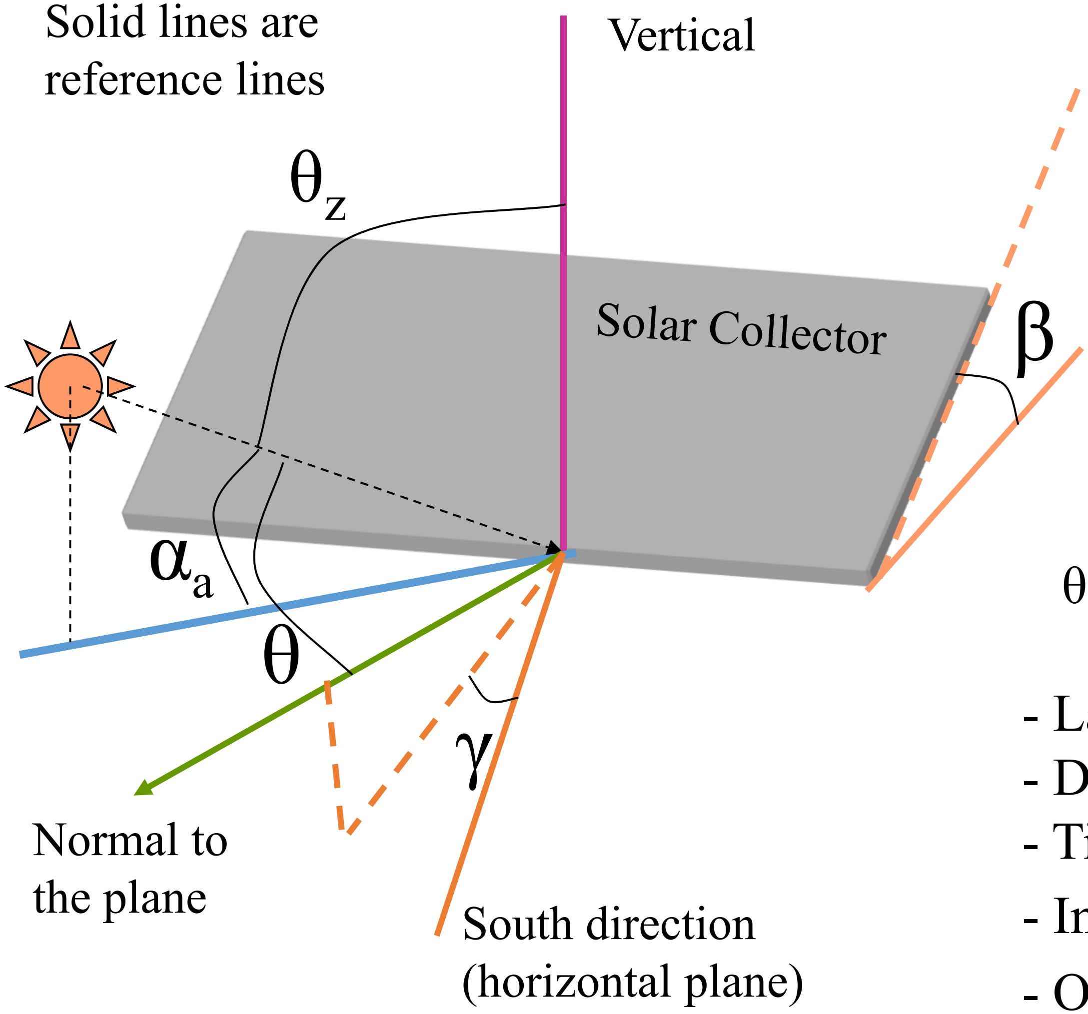
- The amount of radiation falling on a collector surface depends on the angle between the sun ray and perpendicular to collector

$$I = I_b \cos\theta$$

- Generally the collector should be perpendicular (θ=0^o) to the solar radiation at all times

- But this requires continuous sun tracking

Solar radiation geometry



The parameters that can affect the direction of light reaching a surface ?

θ is affected by five parameters

- Latitude of location (φ)
- Day of year (δ)
- Time of the day (w)
- Inclination of surface (β)
- Orientation in horizontal plane (γ)

Angles related to solar geometry

Latitude (φ) – angle of a location on earth w.r.t. to equatorial plane
Surface azimuth angle (+90° to -90°, +ve in the north)

Surface azimuth angle (γ) – angle between surface normal and south direction in horizontal plane, (+180° to -180°, +ve in the east of south)

Hour angle (ω) – angular measure of time w.r.t. noon (LAT), 15° per hour, (+180° to -180°, +ve in the morning)

Surface slope (β) – Angle of the surface w.r.t horizontal plane (0 to 180°)

Declination angle (δ) – Angle made by line joining center of the sun and the earth w.r.t to equatorial plane (+23.45° to -23.45°)

Angle of Sun rays on collector

Incidence angle of rays on collector (w.r.t. to collector normal)

$$\cos\theta = \sin\phi(\sin\delta\cos\beta + \cos\delta\cos\gamma\cos\omega\sin\beta) \\ + \cos\phi(\cos\delta\cos\omega\cos\beta - \sin\delta\cos\gamma\sin\beta) \\ + \cos\delta\sin\gamma\sin\omega\sin\beta$$

Case-1: i.e. $\beta = 0^o$. Thus, for the horizontal surface, then :

$$\cos\theta = \sin\phi\sin\delta + \cos\phi\cos\delta\cos\omega = \cos\theta_z$$

Case-2: $\gamma = 0^o$, collector facing due south

$$\cos\theta = \sin\delta\sin(\phi - \beta) + \cos\delta\cos\omega\cos(\phi - \beta)$$

Angle of Sun rays on collector

What will be the angle of incidence on vertical surface facing due South?

Local apparent time (LAT)

Time at which the Sun becomes overhead at a given location

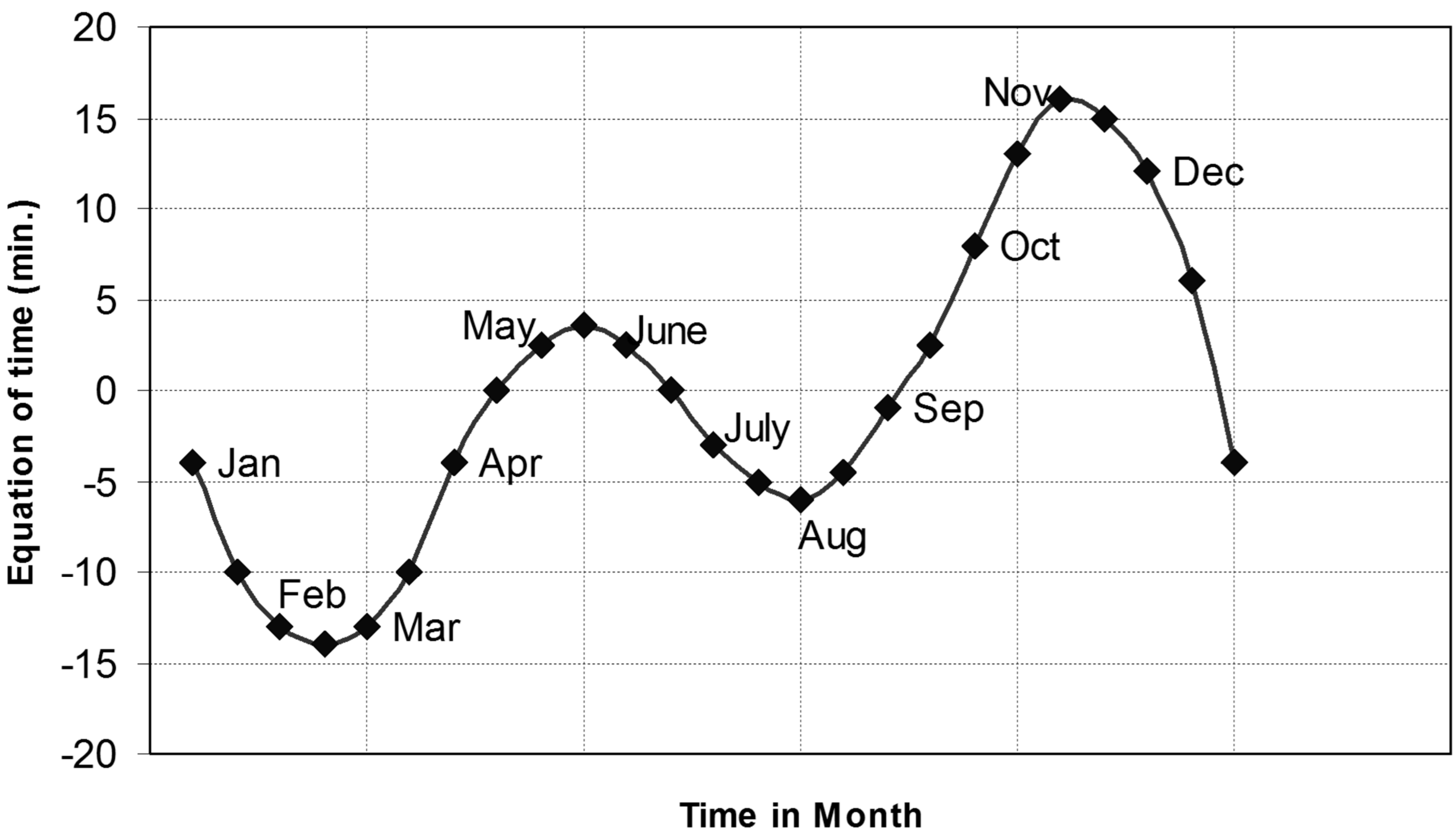
- Normally the standard time for a country is based on a noon (overhead Sun position) at a particular longitude

- Correction in the real noon time by considering the difference in the longitude w.r.t. standard longitude of that country, **1° longitude difference = 4 min.**

$$LAT = T_{local} + \frac{1}{15}(Long_{st} - Long_{local}) + \frac{1}{60} Eq.\ of\ time$$

Equation of time correction

- Due variation in earth's orbit and its speed of revolution
- Based on experimental observations



Local apparent time (LAT)

Local apparent time,

$$LAT = IST - 4\left(\psi_{STD} - \psi_{L}\right) + E$$

$$E = 9.87\sin 2B - 7.53\cos B - 1.5\sin B$$

$$B = \left(n - 81\right)\frac{360}{364}$$

Determine the local apparent time (LAT) corresponding to 1430h at Mumbai (19°07' N, 72°51' E) on July 1. In India, standard time is based on 82.5°E.

Local apparent time (LAT)

Local apparent time,

$$LAT = IST - 4\left(\psi_{STD} - \psi_L\right) + E$$

$$E = 9.87\sin 2B - 7.53\cos B - 1.5\sin B$$

$$B = \left(n - 81\right)\frac{360}{364}$$

n = 182; B = 99.89; E = -3.524

Local apparent time = 1430h – 4(82.50-72.85)minutes + (-3.524) min

=1430h – 38.6minutes-3.5minutes

=1430h-42.1minutes

=13h47.9min = 1348h

Estimation of sunrise and sunset time, day legth

Terminology

Day number of the year	n	number
Latitude	$\varnothing$	degrees
Longitude	Ψ	degrees
Elevation	E_L	km
Declination angle	δ	degrees
Slope of the collector	β	Degrees
Hour angle	ω_s	Degrees
Number of sunshine hours	N	Hours
Maximum number of sunshine hours	S_{max}	Hours
Monthly average daily global radiation		kW/m^2-day
Monthly average daily diffuse radiation		kW/m^2-day
Total daily radiation falling on a tilted surface		kW/m^2-day

Terminology

Day number of the year, n

n varies from 1 to 365

For example: Jan 1st, n =1, and so on.

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
31	28	31	30	31	30	31	31	30	31	30	31

Declination angle

$$\delta = 23.45 \times \sin\left[\frac{360}{365} \times (284 + n)\right]$$

Sunrise and Sunset time

- Solar incidence angle for horizontal collector

$$\cos\theta = \sin\phi\sin\delta + \cos\phi\cos\delta\cos\omega = \cos\theta_z$$

- ❑ For sunrise as well as for sunset the θ=90^o in above equation

Sunrise and Sunset time

- Hour angle at sunrise or sunset as seen by observer on an horizontal surface due south ($\gamma = 0°$) will be

$$\omega_s = \cos^{-1}\left(-\tan\phi \tan\delta\right)$$

This equation is valid for day under consideration lies between **September 22 and March 21** and location is **northern hemisphere** (due to negative declination).

- If the day of consideration is lies between **March 21 and Sep 22**, the equation is

$$\omega_{st} = \cos^{-1}\left[-\tan\left(\phi - \beta\right)\tan\delta\right]$$

Sunrise and sunset time

- The magnitude of hour angle corresponding to sunrise or sunset for an **inclined surface facing south** (γ = 0°) is

$$|\omega_{st}| = \min\left[\left|\cos^{-1}\left(-\tan\phi\tan\delta\right)\right|, \left|\cos^{-1}\left\{-\tan\left(\phi-\beta\right)\tan\delta\right\}\right|\right]$$

The sunrise for the inclined surface will happen later than horizontal surface for dates between 22 March o 21st Sep**t**
Why??

Day Length

Day length between sunrise to sunset

- S_{max} (day length or maximum number of sunshine hours)

$$S_{max} = \frac{2}{15} \times \omega_s = \frac{2}{15} \cos^{-1}\left(-\tan\phi \times \tan\delta\right)$$

Hour Angle

Calculate the hour angle at sunrise and sunset on June 21 and December 21 for a surface inclined at an angle of 10° and facing due south ($\gamma = 0°$). The surface is located in Mumbai (19°07' N, 72°51' E)

Hour Angle

On **June 21**, δ = 23.45°.

Hour angle at sunrise and sunset, ω_{st}

$$\left|\omega_{st}\right| = \min\left[\left|\cos^{-1}\left(-\tan\phi\tan\delta\right)\right|, \left|\cos^{-1}\left\{-\tan\left(\phi-\beta\right)\tan\delta\right\}\right|\right]$$

$$\left|\omega_{st}\right| = \min\begin{bmatrix}\left|\cos^{-1}\left(-\tan 19.2^\circ \tan 23.45^\circ\right)\right|, \\ \left|\cos^{-1}\left\{-\tan\left(19.12^\circ - 10^\circ\right)\tan 23.45^\circ\right\}\right|\end{bmatrix} = \pm 94.0^\circ$$

On **December 21**, δ = -23.45°.

$\omega_{st} = \pm 81.4^\circ$

Estimation of solar radiation

at any given time, any location

Measurement of Solar Radiation, and Estimation of Solar Radiation

- ❑ Measurement of solar radiation is expensive and time taking
- ❑ Models have been developed to estimate solar radiation at given surface

Estimating solar radiation empirically: Global radiation

Estimation of monthly average global radiation on horizontal surface

$$\frac{H_{ga}}{H_{oa}} = a + b\left(\frac{S_a}{S_{\max a}}\right)$$

Where,

H_{ga} → monthly averaged daily global radiation on a horizontal surface

H_{oa} → monthly averaged extra-terrestrial solar radiation at horizontal surface (at top of atmosphere)

S_a and **$S_{max\ a}$** → monthly averaged daily sunshine hours and maximum possible daily sunshine hours (the day length) at a given location.

a and **b** → constant

Values of constants a and b

Location	a	b
Ahmedabad, India	**0.28**	**0.48**
Atlanta, Gerorgia, USA	**0.38**	**0.26**
Brownsville, Texas, USA	**0.35**	**0.31**
Buuenos Aires, Argentina	**0.26**	**0.50**
Charleston, S. C., USA	**0.48**	**0.09**
Bangalore, India	**0.18**	**0.64**
Hamburg, Germany	**0.22**	**0.57**
Malange, Angola	**0.34**	**0.34**
Miami, Florida, USA	**0.42**	**0.22**
Nagpur, India	**0.27**	**0.50**
New Delhi, India	**0.25**	**0.57**
Nice, France	**0.17**	**0.63**
Pune, India	**0.31**	**0.43**
Rafah, Egypt	**0.36**	**0.35**
Stanleyville, Congo	**0.28**	**0.39**
Tamanrasset, Algeria	**0.30**	**0.43**

- The values of the constant is empirically obtained from know data

Ref: Lof J.A. et al, 1966

Estimation of Extra-terrestrial solar radiation – horizontal surface

H_{oa} is equal to H_o if calculated on following days of month;

January 17, February 16, March 16, April 15, May 15, June 11, July 17, August 16, September 15, October 15, November 14 and December 10.

The monthly averaged daily solar extra-terrestrial radiation

$$H_o = S_t \int \cos\theta dt$$

$$H_o = S\left(1 + 0.033\cos\frac{360n}{365}\right) * \int (\sin\phi \sin\delta + \cos\phi\cos\delta\sin\omega)dt$$

$$dt = \frac{180\omega}{15\pi}$$

$$H_o = \frac{24}{\pi} S\left(1 + 0.033\cos\frac{360n}{365}\right)(\omega_s \sin\phi\sin\delta + \cos\phi\cos\delta\sin\omega_s)$$

S is in W/m^2 the H_o will be $W\text{-}Hour/m^2$

Problem:

Estimate the monthly average daily global radiation on the horizontal surface at Nagpur (21.06N, 79.03E) during month of March if the average sunshine hours per day is 9.2. Assume values for a=0.27 and b=0.50

$$\frac{H_{ga}}{H_{oa}} = a + b\left(\frac{S_a}{S_{\max a}}\right)$$

Given: Φ, a, b, S_a
Required: δ, ω_s, $S_{max,a}$, H_{oa}, n, S

$$H_o = H_{oa} = \frac{24}{\pi} S\left(1 + 0.033\cos\frac{360n}{365}\right)(\omega_s \sin\phi \sin\delta + \cos\phi\cos\delta\sin\omega_s)$$

$$\delta = 23.34\sin\left(\frac{360}{365}(284+75)\right)$$

$$\cos\omega_s = -\tan\phi\tan\delta$$

$$\omega_s = \cos^{-1}(-\tan\phi\tan\delta)$$

Solution:

On March 16, n=75

δ= -2.4177

$$\omega_s = \cos^{-1}(-\tan 21.15 \tan -2.4177)$$
$$\omega_s = 89.0640$$

Day length=11.8752 hr, Sunrise and sunset hours?

$$H_o = \frac{24}{\pi} 1.367 * 3600 \left(1 + 0.033 \cos \frac{360 * 57}{365}\right)(1.5544 \sin 21.15 \sin -2.4177 + \cos 21.15 \cos -2.4177 \sin 89.0640)$$

H_o=34140.2 kJ/m^2-day

H_{ga} = 22442.46 kJ/m^2-day

Monthly averaged daily Diffuse radiation

$$for \quad \omega_s \leq 81.4° and\ 0.3 \leq K_T \leq 0.8$$

$$\frac{H_{da}}{H_{ga}} = 1.391 - 3.560K_T + 4.189K_T^2 - 2.137K_T^3$$

$$for \quad \omega_s > 81.4° and\ 0.3 \leq K_T \leq 0.8$$

$$\frac{H_{da}}{H_{ga}} = 1.311 - 3.022K_T + 3.427K_T^2 - 1.821K_T^3$$

Ref: Erbs et al., 1982

ω_s = sunrise hour angle
K_T = sky monthly averaged clearness index, = H_{ga} / H_{oa}

Typically diffuse radiation is about 10 to 20% of the global radiation on horizontal surface

Monthly averaged daily Diffuse radiation- horizontal surface

Studies on Indian solar radiation data

$$0.3 \le K_T \le 0.7$$

$$\frac{H_{da}}{H_{ga}} = 1.411 - 1.696 K_T$$

One other analysis presented following

$$\frac{H_{da}}{H_{ga}} = 1.354 - 1.570 K_T$$

- K_T = sky monthly averaged clearness index, = H_{ga} / H_{oa}

- Typically diffuse radiation is about 10 to 20% of the global radiation on horizontal surface

- Diffuse radiation component in India is larger than Europe

Radiation on tilted south facing surface

- **Angle of sun rays with β=0**

$$\cos\theta = \sin\phi \sin\delta + \cos\phi \cos\delta \cos\omega = \cos\theta_z$$

- **Angle of sun rays with γ=0**

$$\cos\theta = \sin\delta \sin(\phi - \beta) + \cos\delta \cos\omega \cos(\phi - \beta)$$

- Tilt factor for beam radiation $$r_b = \frac{\cos\theta}{\cos\theta_z}$$

- Tilt factor for diffuse radiation $$r_d = \frac{1+\cos\beta}{2}$$

- Tilt factor for reflected radiation $$r_d = \rho\left(\frac{1-\cos\beta}{2}\right)$$

•ρ is reflectivity of surrounding, 0.1 to 0.2

Radiation on tilted south facing surfaces

- Total radiation falling on tilted surface at <u>any instant</u>

$$I_T = I_b r_b + I_d r_d + (I_b + I_d) r_r$$

$$\frac{I_T}{I_g} = \left(1 - \frac{I_d}{I_g}\right) r_b + \frac{I_d}{I_g} r_d + r_r$$

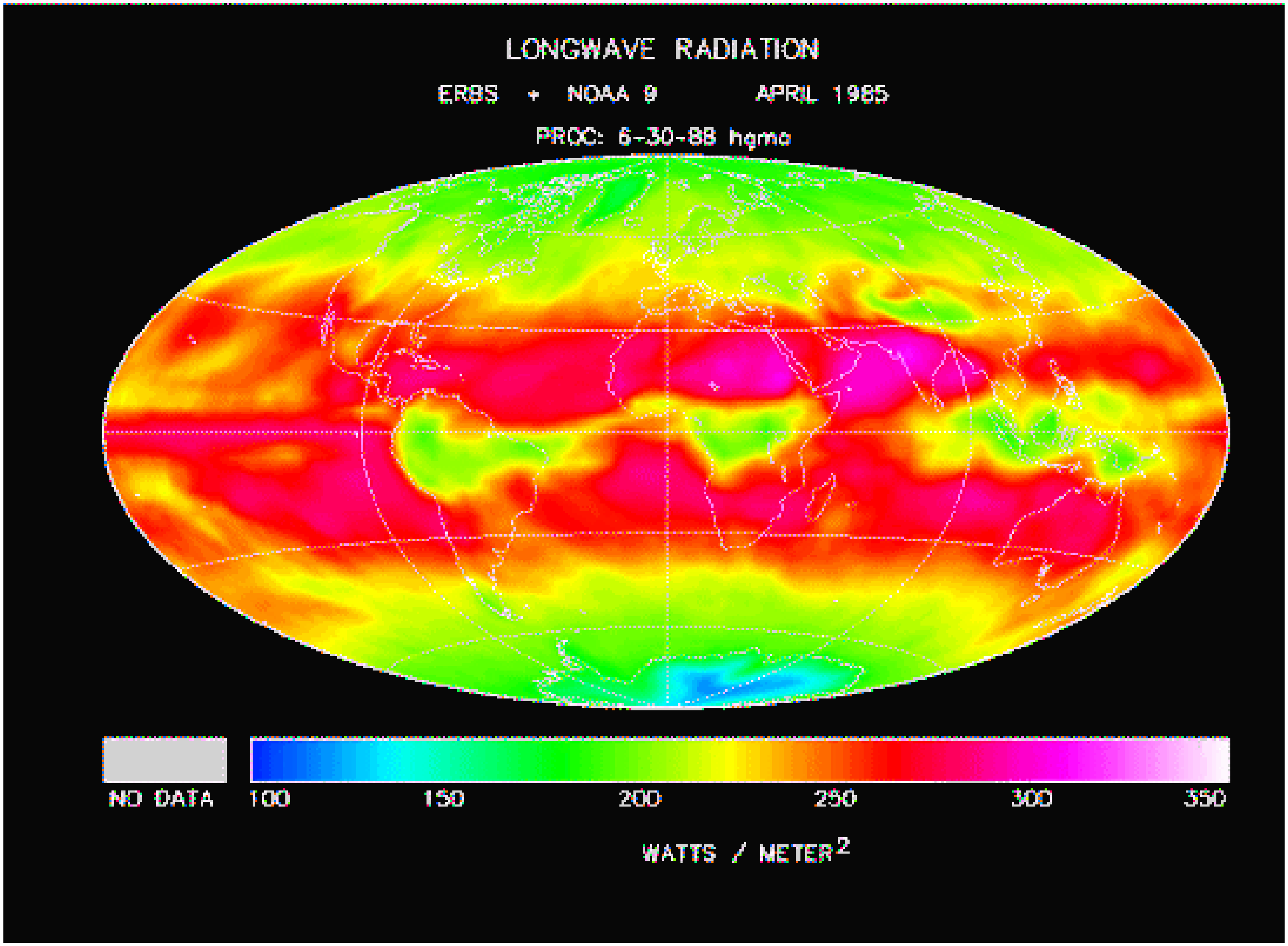
- Total <u>daily</u> radiation falling on south tilted surface, γ=0

$$\frac{H_T}{H_g} = \left(1 - \frac{H_d}{H_g}\right) R_b + \frac{H_d}{H_g} R_d + R_r$$

$$R_b = \frac{\omega_{st} \sin\delta \sin(\phi - \beta) + \cos\delta \cos\omega_{st} \cos(\phi - \beta)}{\omega_s \sin\delta \sin\phi + \cos\delta \cos\phi \cos\omega_s}$$

$$R_d = r_d \quad \& \quad R_r = r_r$$

Global Long-wave Radiation Map



Thank you for your attention

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